Kilburn Junior School

Calculation Policy

Spring 2026

EMBARK FEDERATION

KILBURN JUNIOR SCHOOL

Embark Federation

Policy written by: Adam Barber

Date of last review: March 2026

Date of next review: March 2029

Kilburn Junior School: Calculation Policy 2019

1. RATIONALE

1.1 At Kilburn Junior School, we believe that a solid grounding in number- both the concept of place value and the ability to calculate in all four operations- are essential to understanding mathematics and, indeed, are essential for coping with everyday life with confidence and independence. Mathematics is a universal language that can be used to express our most complex scientific principes.

1.2 Being an effective and confident manipulator of numbers can provide children with a rigorous way of quantifying and interpreting the world in an empirical and scientific manner. This is the basis for success in later life, in an array of disciplines throughout their academic careers and as they move into an ever changing job market. As Galileo said, "[The universe] cannot be read until we have learnt the language and become familiar with the characters in which it is written. It is written in mathematical language, and the letters are triangles, circles and other geometrical figures, without which means it is humanly impossible to comprehend a single word."

1.3 In order to teach children to calculate we must develop within pupils an intrinsic understanding of numbers as *mathematical objects* with numerals being only symbolic representatives of actual quantities. We must also endeavour to extend children's understanding of number to the wider definitions added to the natural numbers over the years (including the concept of 0, positive and negatives (integers), rational numbers and, to some extent, irrational numbers).

1.4 The policy is intended to be read in conjunction with the mathematics policy which illustrates the school's overarching pedagogical approach to the teaching of mathematics across all areas of the curriculum. This, more specific, calculation policy has been written in order to make the strategies and methods outlined in the national curriculum 'context specific' so that a progressive and effective curriculum is delivered to all our pupils.

1.5 Within this policy we aim to cover the following areas:

A. The principles of teaching calculation
B. The essential conceptual understanding underpinning effective calculation
C. The progression of vocabulary and symbols needed to write a variety of expressions
D. The progression and breadth of mental calculation strategies in all four operations
E. The progression and breadth of written calculation strategies for all four operations (including fractions arithmetic and the order of operations)
F. Materials available to support teaching and learning

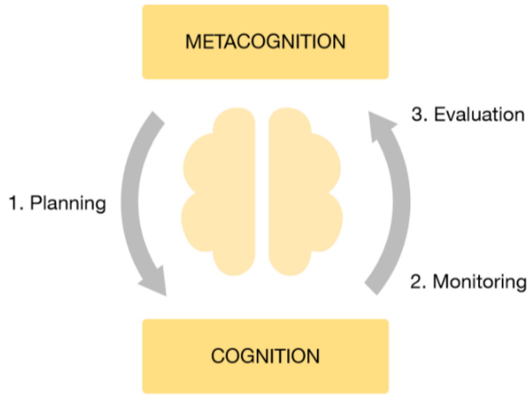
2. PRINCIPLES OF TEACHING CALCULATIONS

3.1 There are several principles relating to the teaching and learning of calculation that we, as a school, believe will enable children to build confidence and efficiency in this area of mathematics.

- **A strong grounding in place value**: This is essential to understanding how to manipulate numerals in order to allow for them to be processed in calculations in the most efficient manner. Without this, learning the times tables, for example, has limited value as these facts cannot then be extrapolated for other calculations. For example, by knowing 6x7=42, we can extend the application of this mathematical fact to answer 60x7, 60x70, 0.6x7 etc...
  It is essential for children to have an understanding of how, symbolically, numbers are represented in the Indo-Arabic numeral system using our positional, decimal numeral system. A useful cross curricular link for

children to consider the efficacy of our current system is to study other numeral systems such as those used by the Romans or the Maya (a basic understanding of the former is stipulated by the National Curriculum).

- **Worked examples**: For children to most efficiently learn, for example the various algorithms associated with the four operation, the study of worked examples has been shown to constitute strongly "guided instruction". The "worked example effect" denotes the effect on learning that the studying of worked examples has on pupils as opposed to the practice of problems. Cognitive Load Theory suggests that working on problems rather than studying worked examples can generate a heavy working memory load that is detrimental to learning especially for novice learners who may lack proper schemas to integrate new information into long-term memory.[1] It is essential therefore that children have extensive exposure to teachers demonstrating and talking through mental and written arithmetic methods as well as children being given time to study written examples (completed, partially completed or with errors). This will allow children to demonstrate the conceptual knowledge and more effectively instil procedural understanding into their long-term memory.
- **Practice and Review**: As the 'Ebbinghaus Forgetting Curve' shows us, information is 'lost' over time when there is no opportunity/attempt to retain it. The forgetting curve demonstrates the age-old adage 'use it or lose it' or in psychological terms, the "transience" of memory, which is the process of 'forgetting' that occurs with the passage of time. Although this could be more accurately described as the lack of ability to recall the information rather that the loss of it from our memories.
- **Teach a range of calculation strategies**: Children need to be taught a variety of strategies and be encouraged to monitor and evaluate the efficiency of these. In doing so, children will not become too reliant on one method for each operation. They will also become critical of whether the methods that they have chosen are efficient in the context and with the numbers they are presented with. In doing so, children will find that the metacognition cycle becomes embedded in their thinking and enables them to select and evaluate strategies so that future calculations will be completed with speed and efficiency. Reflecting on their learning in this way will improve the self-regulation of their methods when calculating independently.



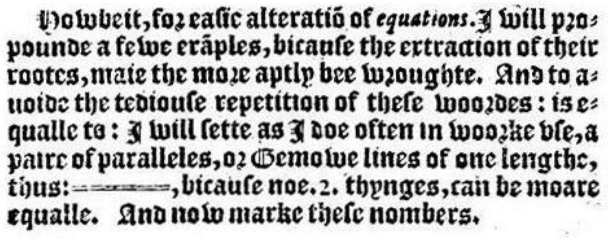
- **Make everyday a mental maths day**: In all mathematics lessons, teachers should allow time for the practice of mental calculation strategies whether these are embedded within the main part of the lesson or carried out separately (on the rare occasion where no link can be made.) There is great merit to devoting class time to counting on or back in different steps from different numbers or to play a multiplication game as a class. The quick recall of number bonds, Teaching should encourage children to work quickly and efficiently in their heads to calculate. Again, teachers need to lead this, talking through how they have completed calculations and encouraging children to think critically about the efficiency of their methods.
- **Ensure children view mental and written calculation as mutually supportive**: Written calculation is a formalised algorithm for ordering the stages of a calculation made up of several mental calculations tracked on paper in a formalised structure. Mental calculation, on the other hand, is not solely completed in the child's head and is often much more efficient when accompanied by informal jottings on paper or through the use of apparatus. Again, teachers need to ensure that they are modelling this for the children and talking through the mental processes involved at each stage of the calculation.
- **Develop a concept of bridging through 10 rather than simply counting on**: Studies, particulary those conducted on exchanges with teachers from Shanghai, have shown that in order to ensure efficiency with mental calculations (for addition and subtraction), children need to develop an understanding of the importance of multiples of 10 and bridging through these. For example, 9+6 could best be completed by younger children by considering the calculation to be 9+1+5 which becomes 10+5. This method extrapolated can increase the fluency of calculations considerably with integers as well as fractions and decimals.[2]

---

[1] *Why Minimal Guidance During Instruction Does Not Work: An Analysis of the Failure of Constructivist , Discovery, Problem-Based, Experiential, and Inquiry-Based Teaching.* EDUCATIONAL PSYCHOLOGIST 41(2) 2006 (Kirscher, Sweller, Clark) p 80

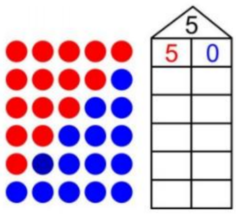
[2] Calculation Guidance: NCETM (2015)

- **Develop understanding of the = symbol**: Children can often have a misunderstanding of the 'equals' symbol and consider it to be an indicator to 'write an answer here'. This basic misconception can lead children to be confused by representations such as the following: 3 + 4 = 6 + 1. This is term can lead to later struggles with even the most simple algebraic expressions such as: 3y=18.
Without an understanding of the = symbol as an assertion of equivalence, a balance of what is on one side with the other, given the calculation 3 + 4 = □ + 1, their natural inclination might be to write a 7 in the empty box. In order to develop the children's understanding of the = symbol as equivalence, we must vary the position of the symbol and include empty box problems as standard practice.


Howbeit, for easie alteratiō of *equations*. I will propounde a fewe exāples, bicause the extraction of their rootes, maie the more aptly bee wroughte. And to auoide the tediouse repetition of these woordes: is equalle to: I will sette as I doe often in woorke use, a paire of paralleles, or Gemowe lines of one lengthe, thus: ═══════, bicause noe.2. thynges, can be moare equalle. And now marke these nombers.

Welsh mathematican, Robert Recorde's introduction of the "=" symbol from his book The Whetstone of Witte (1557).

- **Develop a concept of inequalities and the associated symbolic representations**: To help young children develop their understanding of equality, they also need to develop an understanding of inequality. Incorporating both equality and inequality into examples and exercises can help children develop their conceptual understanding. For example, in this empty box problem children have to decide whether the missing symbol is <, = or >: 5 + 7 □ 5 + 6.
Asking children to decide if number sentences are true or false also helps develop mathematical reasoning. For example, in discussing this statement 4 + 6 + 8 > 3 + 7 + 9 a child might reason that "4 plus 6 and 3 plus 7 are both 10. But 8 is less than 9 therefore 4 + 6 + 8 must be less than 3 + 7 + 9, not more than 3 + 7 + 9".
- **Engage children in discussion and in verbal explanation** when they explain their methods. In order to effectively teach conceptual understanding as well as procedural understanding, it is necessary for children to be able to articulate the stages of the the methods they are employing and the reasons for their choices be they mental or written strategies. In doing so, children will be need to go through the process of planning, monitoring and evaluating their learning. This will enable them to more effectively choose from a range of strategies in the future.
- **Highlight Common Misconceptions and Non-Examples**: As with any quality instruction, highlighting misconceptions is essential for a number of reasons: firstly so that children can learn from mistakes and improve procedural and conceptual understanding; secondly, so that misconceptions are not committed to their long-term memory and thirdly so that complicated problems can be anticipated by the children who already have a method with which to tackle them. For example: where exchanging will need to take place from two columns over in subtraction or when trying to divide fractions by whole numbers without first finding the reciprocal of the divisor. When teaching mental calculation strategies, it is vital for the children to develop an understanding of where the specific strategy will be efficient and where it will not. In order to do this, examples need to be routinely studied alongside non-examples. By carefully selecting questions during discrete mental calculation lessons, pupils can be encouraged to consider if the taught strategy is the most efficient or where another might more readily lead to a solution.
- **Expose Mathematical Structure**: Developing instant recall alongside conceptual understanding of number bonds and other such mathematical facts is very important. For example when teaching number bonds to 5 the image to the right lends itself to seeing pattern and working systematically. Children can connect one number fact to another and be certain when they have found all the bonds to 5. (This example is taken from a Shanghai textbook used with 6/7 year olds.) Using other structured models such as tens frames, Cuisenaire rods, part whole models or bar models can help children to reason about mathematical relationships in calculations.



3. CONCEPTUAL KNOWLEDGE

3.1 There are several mathematical laws that adults seldom have to devote any mental attention to as they become automatic after many years of calculating mentally and in more formal written methods. These mathematical laws- whilst seemingly intuitive for confident mathematicians- will need reinforcing for children regularly, even for more confident mathematicians. Children will need to understand the following concepts in they are to become efficient in calculating both mentally and in written form.

The following are essential mathematical concepts that must be understood by children (in principle, if not in name):

- **The Commutative Law**

  When we add or multiply, the order of the numbers will not affect the outcome no matter how many numbers are in the calculation. 5 + 4 = 9 and 4 + 5 = 9. The 5 and the 4 can be switched in addition. Likewise, 9x2 = 18 and 2 x 9 = 18. The 9 and the 2 can be switched in multiplication. "Switching" or "changing" the order of numbers is called "commuting". When we change the order of the numbers, we have applied the "Commutative Law". In an addition problem, it is referred to as the "Commutative Law of Addition and in multiplication, it is the "Commutative Law of Multiplication."

  The Commutative Law does not work for either subtraction or division. These are said to be "non-commutative" The order of the numbers will affect the outcome. 80 – 25 will yield a different outcome to 25 – 80.

- **The Associative Law**

  The Associative Law can be thought of as an extension to the Commutative Law. When we have more than two numbers to add at a time we can again manipulate the order of them in order to make them easier to calculate with. The assosciative law moves parentheses rather than alter the order of the calculation.

  For example: consider 3 + 10 + 2

  You could first add 3 and 10 to get 13. Then add the result to 2 and obtain 15.

  (3 + 10) + 2 =

  (13) + 2 = 15

  Or you could first add 10 and 2 to get 12. Then add the result to 3 to get 15.

  3 + (10 + 2) = 3 + (12) = 15

  In both cases, we obtained the same answer.

  (3 + 10) + 2 = 3 + (10 + 2)

  Notice that the numbers: 3, 10, and 2 did not move. What did move was the parentheses. In the first case, the parentheses were associated with the first two numbers 3 and 10. The second time we tried the problem, they were placed around (associated with) the 10 and 2. This may seem to simply describe the Commutative Law again but in fact is distinct and will have greater bearing on the children as they begin to work with basic algebra in year 6 and then into the next phase of their learning. The Associative Law allows you to move parentheses as long as the numbers do not move. As with the commutative law, this will work only for addition and multiplication.

- **The Distributive Law**

  The Distributive Law says that multiplying a number by a group of numbers added together is the same as doing each multiplication separately.

  Example: 3 × (2 + 4) = 3×2 + 3×4

  So the "3" can be "distributed" across the "2+4" into 3 times 2 and 3 times 4.

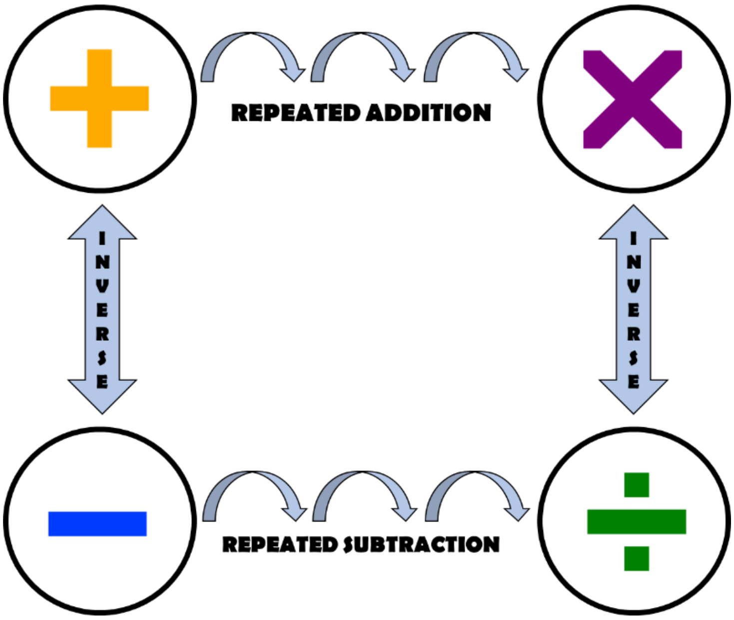
  Likewise:

  (4 + 5) × 3 = (4 × 3) + (5 × 3)

  This feature can be very useful in mental calculations both as a way of manipulating numbers to make them easier for children to calculate mentally or for checking calculations completed in a different manner.

- **The Relationship Between the Four Operations**

  Addition, subtraction, multiplication and division should not be seen as four disparate actions but should be taught as interrelated. Where one operation undoes (cancels out) another, they are said to be **inverse**. This inverse relationship obviously exists between addition and subtraction, multiplication and division, but also between square numbers and their square roots and cube numbers and their cube roots etc… A thorough understanding and use of this inverse relationship between the operations can help children to become much better at checking calculations after they have arrived at an answer. This knowledge is also vital later in their academic career as they begin to solve algebraic equations or deal with functions.

  Secondly, an understanding of multiplication as repeated addition can help manipulate numbers in order to make larger calculations more manageable. Likewise, an understanding of division as repeated subtraction is essential to accessing division calculations for KS1 and early KS2.

  For the calculation 143 x 3, children may find that 143 + 143 + 143 is an easier calculation to initially answer the problem, or indeed could use this repeated addition calculation as method of validating their multiplication rather than complete 429 ÷ 3. Different children will prefer different methods and highlighting the different relationships between the four operations will allow them to manipulate calculations in order to suit them.

  A useful link between multiplication and addition allows children to work out new facts from facts that they already know. For example, the child who can work out the answer to 8 × 6 (six eights) by recalling 8 × 5 (five eights) and then adding 8 will, through regular use of this strategy, become more familiar with the fact that 8 × 6 is 48.

4. SPOKEN LANGUAGE

4.1 Children should not be expected to be able to formulate coherent mathematical statements without first having had these modelled to them.

4.2 When asking or answering questions, children should be taught to answer in full sentences rather than with single words or numbers. For example, a child asked what is 20 x 6 should be encouraged to answer with "20 multiplied by 6 equals 120" rather than just "120".

4.3 Likewise, as teachers, we should ensure that we are using the full variety of vocabulary when asking questions and not simply use the same words over and over. The above question could be asked "what is 20 multiplied by 6", "what are 20 groups of 6", "what is the product of 20 and 6", "what is 20 times by 6" etc…

4.4 The variety and scope of the vocabulary that it is essential to try to instil in the children is set out clearly in the NCETM document *Mathematics glossary for teachers in Key Stages 1 to 3*.

4.5 The vocabulary that children will be focussing on for each lesson will be displayed alongside the intended learning outcomes for the lesson[3] and other appropriate locations for the children to access and to use to support their spoken interactions with the teachers and their peers. The teaching staff will endeavour to make this vocabulary as specific and as accurate as possible.

[3] See appendix 1 of the Mathematic policy.

5. CALCULATORS

5.1 The four basic mathematical operations- addition, subtraction, multiplication, and division- have application even in the most advanced mathematical theories. Thus, mastering them is one of the keys to progressing in our understanding of mathematics. Electronic calculators have made these (and other) operations simple to perform, but these devices can also create a dependency that makes really understanding mathematics quite difficult. Calculators can be a handy tool for checking answers, but if children rely too heavily on them, we may deprive them of the kind of rigorous mental exercises that will help them to not just to do maths, but to fully understand the conceptual underpinning of what they are undertaking.

5.2 Calculators should not be used as a substitute for good written and mental arithmetic. They should therefore be introduced when the intention is to support pupils' conceptual understanding and exploration of more complex number problems or to reduce cognitive overload when exploring concepts where arithmetic is not the focus of the lesson. Teachers should use their judgement about when ICT tools should be used to enhance or support teaching and learning.

6. ASSESSMENT OF CALCULATION

6.1 Assessment should be used not only to track pupils' learning but also to provide teachers at Kilburn Junior School with information about what pupils do and do not know. This information allows teachers to adapt their teaching so it builds on pupils' existing knowledge, addresses their weaknesses, and focuses on the next steps that they need in order to make progress.

6.2 Teachers' knowledge of pupils' strengths and weaknesses is used to inform the planning of future lessons and the focus of targeted support.

6.3 A variety of assessment methods are used to build a picture of the children's learning. Formal tests can be useful here, although assessment can also be based on evidence from low-stakes class assessments, informal observation of pupils, or discussions with them about mathematics. A schedule of the different focuses and appropriate tests that we conduct to assess children's understanding and plan for future intervention can be found in the document ***Assessment at Kilburn Junior School.***

6.4 Pupils' work will be marked in line with the ***Marking Policy*** and will model how corrections should be made, giving pupils a chance to learn from their misconceptions or incorrect methods.

6.5 Summative assessments are made at least once per term, six times per academic year and logged on Arbor. These teacher assessments are based on the evidence from formative and summative assessments made by the class teacher. The mathematics subject co-ordinator will organise moderation and standardisation activities to ensure the accuracy of the assessments being made by the class teachers.

6.6 The use of the progressive arithmetic tests for each year group and the question level analysis documents allow for staff to easily track individual and group progress with written strategies. These have also proven to be an invaluable tool for ensuring children are given regular practice at age-appropriate arithmetic questions.

7. PARENTAL SUPPORT AND HOMEWORK

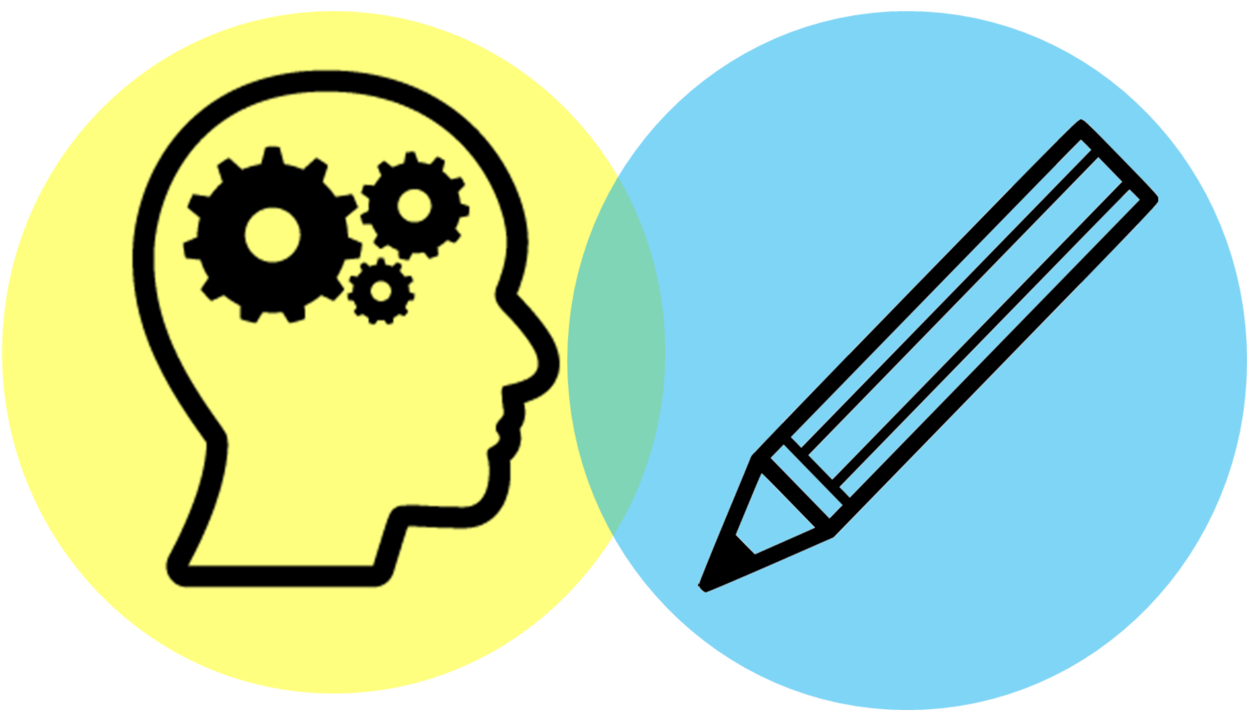
7.1 We recognise that parents make a significant difference to the progress of pupils in maths and encourage this essential partnership.

7.2 Homework is used for the following purposes:

- To practice a skill.
- To learn something by rote such as times tables and formulae.
- To revise for a test.
- To explore a mathematical problem or question.
- To research a topic.

Kilburn Junior School

Mental and Written Calculation Guidance



EMBARK FEDERATION

Kilburn Junior School

Mental Calculation Guidance

EMBARK
FEDERATION

Mental Calculation

Progression of Skills

	Recall	Mental calculation skills	Mental methods or strategies
YEAR 1	• number pairs with a total of 10, e.g. 3 + 7, or what to add to a single-digit number to make 10, e.g. 3 + □ = 10 • addition facts for totals to at least 5, e.g. 2 + 3, 4 + 3 • addition doubles for all numbers to at least 10, e.g. 8 + 8	• add or subtract a pair of single-digit numbers, e.g. 4 + 5, 8 – 3 • add or subtract a single-digit number to or from a teens number, e.g. 13 + 5, 17 – 3 • add or subtract a single-digit to or from 10, and add a multiple of 10 to a single-digit number, e.g. 10 + 7, 7 + 30 • add near doubles, e.g. 6 + 7	• reorder numbers when adding, e.g. put the larger number first • count on or back in ones, twos or tens • partition small numbers, e.g. 8 + 3 = 8 + 2 + 1 • partition and combine tens and ones • partition: double and adjust, e.g. 5 + 6 = 5 + 5 + 1
YEAR 2	• addition and subtraction facts for all numbers up to at least 10, e.g. 3 + 4, 8 – 5 • number pairs with totals to 20 • all pairs of multiples of 10 with totals up to 100, e.g. 30 + 70, or 60 + □ = 100 • what must be added to any two-digit number to make the next multiple of 10, e.g. 52 + □ = 60 • addition doubles for all numbers to 20, e.g. 17 + 17 and multiples of 10 to 50, e.g. 40 + 40	• add or subtract a pair of single-digit numbers, including crossing 10, • e.g. 5 + 8, 12 – 7 • add any single-digit number to or from a multiple of 10, e.g. 60 + 5 • subtract any single-digit number from a multiple of 10, e.g. 80 – 7 • add or subtract a single-digit number to or from a two-digit number, including crossing the tens boundary, e.g. 23 + 5, 57 – 3, then 28 + 5, 52 – 7 • add or subtract a multiple of 10 to or from any two-digit number, e.g. 27 + 60, 72 – 50 • add 9, 19, 29, … or 11, 21, • 31, … • add near doubles, e.g. 13 + 14, 39 + 40	• reorder numbers when adding • partition: bridge through 10 and multiples of 10 when adding and subtracting • partition and combine multiples of tens and ones • use knowledge of pairs making 10 • partition: count on in tens and ones to find the total • partition: count on or back in tens and ones to find the difference • partition: add a multiple of 10 and adjust by 1 • partition: double and adjust

Mental Calculation

Progression of Skills

	Recall	Mental calculation skills:	Mental methods or strategies
YEAR 3	• addition and subtraction facts for all numbers to 20, e.g. 9 + 8, 17 – 9, drawing on knowledge of inverse operations • sums and differences of multiples of 10, e.g. 50 + 80, 120 – 90 • pairs of two-digit numbers with a total of 100, e.g. 32 + 68, or 32 + □ = 100 • addition doubles for multiples of 10 to 100, e.g. 90 + 90	• add and subtract groups of small numbers, e.g. 5 – 3 + 2 • add or subtract a two-digit number to or from a multiple of 10, e.g. 50 + 38, 90 – 27 • add and subtract two-digit numbers e.g. 34 + 65, 68 – 35 • add near doubles, e.g. 18 + 16, 60 + 70	• reorder numbers when adding • identify pairs totalling 10 or multiples of 10 • partition: add tens and ones separately, then recombine • partition: count on in tens and ones to find the total • partition: count on or back in tens and ones to find the difference • partition: add or subtract 10 or 20 and adjust • partition: double and adjust • partition: count on or back in minutes and hours, bridging through 60 (analogue times)
YEAR 4	• sums and differences of pairs of multiples of 10, 100 or 1000 • addition doubles of numbers 1 to 100, e.g. 38 + 38, and the corresponding halves • what must be added to any three-digit number to make the next multiple of 100, e.g. 521 + □ = 600 • pairs of fractions that total 1	• add or subtract any pair of two-digit numbers, including crossing the tens and 100 boundary, e.g. 47 + 58, • 91 – 35 • add or subtract a near multiple of 10, e.g. 56 + 29, 86 – 38 • add near doubles of two-digit numbers, e.g. 38 + 37 • add or subtract two-digit or three-digit multiples of 10, e.g. 120 – 40, 140 + 150, 370 – 180	• count on or back in hundreds, tens and ones • partition: add tens and ones separately, then recombine • partition: subtract tens and then ones, e.g. subtracting 27 by subtracting 20 then 7 • subtract by counting up from the smaller to the larger number • partition: add or subtract a multiple of 10 and adjust, • e.g. 56 + 29 = 56 + 30 – 1, or 86 – 38 = 86 – 40 + 2 • use knowledge of place value and related calculations, e.g. work out 140 + 150 = 290 using 14 + 15 = 29 • partition: count on or back in minutes and hours, bridging through 60 (analogue and digital)

Mental Calculation

Progression of Skills

	Recall	Mental calculation skills:	Mental methods or strategies
YEAR 5	• sums and differences of decimals, e.g. 6.5 + 2.7, 7.8 – 1.3 • doubles and halves of decimals, e.g. half of 5.6, double 3.4 • what must be added to any four-digit number to make the next multiple of 1000, e.g. 4087 + □ = 5000 • what must be added to a decimal with units and tenths to make the next whole number, e.g. 7.2 + □ = 8	• add or subtract a pair of two-digit numbers or three-digit multiples of 10, e.g. 38 + 86, 620 – 380, 350+ 360 • add or subtract a near multiple of 10 or 100 to any two-digit or three-digit number, e.g. 235 + 198 • find the difference between near multiples of 100, e.g. 607 – 588, or of 1000, e.g. 6070 – 4087 • add or subtract any pairs of decimal fractions each with units and tenths, e.g. 5.7 + 2.5, 6.3 – 4.8	• count on or back in hundreds, tens, ones and tenths • artition: add hundreds, tens or ones separately, then recombine • subtract by counting up from the smaller to the larger number • add or subtract a multiple of 10 or 100 and adjust • partition: double and adjust • use knowledge of place value and related calculations, e.g. 6.3 – 4.8 using 63 – 48 • partition: count on or back in minutes and hours, bridging through 60 (analogue and digital times)
YEAR 6	• addition and subtraction facts for multiples of 10 to 1000 and decimal numbers with one decimal place, e.g. 650 + □ = 930, □ – 1.4 = 2.5 • what must be added to a decimal with units, tenths and hundredths to make the next whole number, e.g. 7.26 + □ = 8	• add or subtract pairs of decimals with units, tenths or hundredths, e.g. 0.7 + 3.38 • find doubles of decimals each with units and tenths, e.g. 1.6 + 1.6 • add near doubles of decimals, e.g. 2.5 + 2.6 • add or subtract a decimal with units and tenths, that is nearly a whole number, • e.g. 4.3 + 2.9, 6.5 – 3.8	• count on or back in hundreds, tens, ones, tenths and hundredths • use knowledge of place value and related calculations, e.g. 680 + 430, 6.8 + 4.3, 0.68 + 0.43 can all be worked out using the related calculation 68 + 43 • use knowledge of place value and of doubles of two-digit whole numbers • partition: double and adjust • partition: add or subtract a whole number and adjust, e.g. 4.3 + 2.9 = 4.3 + 3 – 0.1, 6.5 – 3.8 = 6.5 – 4 + 0.2 • partition: count on or back in minutes and hours, bridging through 60 (analogue and digital)

Mental Calculation

Progression of Skills

	Recall	Mental calculation skills	Mental methods or strategies
YEAR 1	• doubles of all numbers to 10, e.g. double 6 • odd and even numbers to 20	• count on from and back to zero in ones, twos, fives or tens	• use patterns of last digits, e.g. 0 and 5 when counting in fives
YEAR 2	• doubles of all numbers to 20, e.g. double 13, and corresponding halves • doubles of multiples of 10 to 50, e.g. double 40, and corresponding halves • multiplication facts for the 2, 5 and 10 times-tables, and corresponding division facts • odd and even numbers to 100	• double any multiple of 5 up to 50, e.g. double 35 • halve any multiple of 10 up to 100, e.g. halve 90 • find half of even numbers to 40 • find the total number of objects when they are organised into groups of 2, 5 or 10	• partition: double the tens and ones separately, then recombine • use knowledge that halving is the inverse of doubling and that doubling is equivalent to multiplying by two • use knowledge of multiplication facts from the 2, 5 and 10 times-tables, e.g. recognise that there are 15 objects altogether because there are three groups of five
YEAR 3	• multiplication facts for the 2, 3, 4, 5, 6 and 10 times-tables, and corresponding division facts • doubles of multiples of 10 to 100, e.g. double 90, and corresponding halves	• double any multiple of 5 up to 100, e.g. double 35 • halve any multiple of 10 up to 200, e.g. halve 170 • multiply one-digit or two-digit numbers by 10 or 100, e.g. 7 × 100, 46 × 10, 54 x 100 • find unit fractions of numbers and quantities involving halves, thirds, quarters, fifths and tenths	• partition: when doubling, double the tens and ones separately, then recombine • partition: when halving, halve the tens and ones separately, then recombine • use knowledge that halving and doubling are inverse operations • recognise that finding a unit fraction is equivalent to dividing by the denominator and use knowledge of division facts • recognise that when multiplying by 10 or 100 the digits move one or two places to the left and zero is used as a place holder

Mental Calculation

Progression of Skills

	Recall	Mental calculation skills:	Mental methods or strategies
YEAR 4	• multiplication facts to 10 × 10 and the corresponding division facts • doubles of numbers 1 to 100, e.g. double 58, and corresponding halves • doubles of multiples of 10 and 100 and corresponding halves • fraction and decimal equivalents of one-half, quarters, tenths and hundredths, e.g. 310 is 0.3 and 3100 is 0.03 • factor pairs for known multiplication facts	• double any two-digit number, e.g. double 39 • double any multiple of 10 or 100, e.g. double 340, double 800, and halve the corresponding multiples of 10 and 100 • halve any even number to 200 • find unit fractions and simple non-unit fractions of numbers and quantities, • e.g. 38 of 24 • multiply and divide numbers to 1000 by 10 and then 100 (whole-number answers), e.g. 325 × 10, 42 × 100, 120 ÷ 10, 600 ÷ 100, 850 ÷ 10 • multiply a multiple of 10 to 100 by a single-digit number, e.g. 40 × 3 • multiply numbers to 20 by a single-digit, e.g. 17 × 3 • identify the remainder when dividing by 2, 5 or 10 • give the factor pair associated with a multiplication fact, e.g. identify that if 2 x 3 = 6 then 6 has the factor pair 2 and 3	• partition: double or halve the tens and ones separately, then recombine • use understanding that when a number is multiplied or divided by 10 or 100, its digits move one or two places to the left or the right and zero is used as a place holder • use knowledge of multiplication facts and place value, e.g. 7 x 8 = 56 to find 70 x 8, 7 x 80 • use partitioning and the distributive law to multiply, e.g. 13 × 4 = (10 + 3) × 4 = (10 × 4) + (3 × 4) = 40 + 12 = 52

Mental Calculation

Progression of Skills

	Recall	Mental calculation skills	Mental methods or strategies
YEAR 5	• squares to 10 × 10 • division facts corresponding to tables up to 10 × 10, and the related unit fractions, e.g. 7 × 9 = 63 so one-ninth of 63 is 7 and one-seventh of 63 is 9 • percentage equivalents of one-half, one-quarter, three-quarters, tenths and hundredths • factor pairs to 100	• multiply and divide two-digit numbers by 4 or 8, e.g. 26 × 4, 96 ÷ 8 • multiply two-digit numbers by 5 or 20, e.g. 320 × 5, 14 × 20 • multiply by 25 or 50, e.g. 48 × 25, 32 × 50 • double three-digit multiples of 10 to 500, e.g. 380 × 2, and find the corresponding halves, e.g. 760 ÷ 2 • find the remainder after dividing a two-digit number by a single-digit number, e.g. 27 ÷ 4 = 6 R 3 • multiply and divide whole numbers and decimals by 10, 100 or 1000, e.g. 4.3 × 10, 0.75 × 100, 25 ÷ 10, 673 ÷ 100, 74 ÷ 100 • multiply pairs of multiples of 10, e.g. 60 × 30, and a multiple of 100 by a single digit number, e.g. 900 × 8 • divide a multiple of 10 by a single-digit number (whole number answers) e.g. 80 ÷ 4, 270 ÷ 3 • find fractions of whole numbers or quantities, e.g. • 23 of 27, 45 of 70 kg • find 50%, 25% or 10% of whole numbers or quantities, e.g. 25% of 20 kg, 10% of £80 • find factor pairs for numbers to 100, e.g. 30 has the factor pairs 1 × 30, 2 × 15, 3 × 10 and 5 × 6	• multiply or divide by 4 or 8 by repeated doubling or halving • form an equivalent calculation, e.g. to multiply by 5, multiply by 10, then halve; to multiply by 20, double, then multiply by 10 • use knowledge of doubles/halves and understanding of place value, e.g. when multiplying by 50 multiply by 100 and divide by 2 • use knowledge of division facts, e.g. when carrying out a division to find a remainder • use understanding that when a number is multiplied or divided by 10 or 100, its digits move one or two places to the left or the right relative to the decimal point, and zero is used as a place holder • use knowledge of multiplication and division facts and understanding of place value, e.g. when calculating with multiples of 10 • use knowledge of equivalence between fractions and percentages, e.g. to find 50%, 25% and 10% • use knowledge of multiplication and division facts to find factor pairs

Mental Calculation

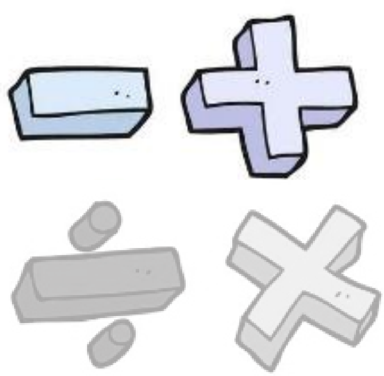
Progression of Skills

	Recall	Mental calculation skills	Mental methods or strategies
YEAR 6	• squares to 12 × 12 • squares of the corresponding multiples of 10 • prime numbers less than 100 • equivalent fractions, decimals and percentages for hundredths, e.g. 35% is equivalent to 0.35 or $\frac{35}{100}$	• multiply pairs of two-digit and single-digit numbers, e.g. 28 × 3 • divide a two-digit number by a single-digit number, e.g. 68 ÷ 4 • divide by 25 or 50, e.g. 480 ÷ 25, 3200 ÷ 50 • double decimals with units and tenths, e.g. double 7.6, and find the corresponding halves, e.g. half of 15.2 • multiply pairs of multiples of 10 and 100, e.g. 50 × 30, 600 × 20 • divide multiples of 100 by a multiple of 10 or 100 (whole number answers), e.g. 600 ÷ 20, 800 ÷ 400, 2100 ÷ 300 • multiply and divide two-digit decimals such as 0.8 × 7, 4.8 ÷ 6 • find 10% or multiples of 10%, of whole numbers and quantities, e.g. 30% of 50 ml, 40% of £30, 70% of 200 g • simplify fractions by cancelling • scale up and down using known facts, e.g. given that three oranges cost 24p, find the cost of four oranges • identify numbers with odd and even numbers of factors and no factor pairs other than 1 and themselves	• partition: use partitioning and the distributive law to divide tens and ones separately, e.g. 92 ÷ 4 = (80 + 12) ÷ 4 = 20 + 3 = 23 • form an equivalent calculation, e.g. to divide by 25, divide by 100, then multiply by 4; to divide by 50, divide by 100, then double • use knowledge of the equivalence between fractions and percentages and the relationship between fractions and division • recognise how to scale up or down using multiplication and division, e.g. if three oranges cost 24p: one orange costs 24 ÷ 3 = 8p four oranges cost 8 × 4 = 32p • Use knowledge of multiplication and division facts to identify factor pairs and numbers with only two factors

MENTAL CALCULATION

ADDITION & SUBTRACTION

- Counting forwards and backwards
- Reordering
- Partitioning: counting on or back
- Partitioning: bridging a multiple of 10
- Partitioning: compensating
- Partitioning: using near doubles
- Partitioning: bridging through numbers other than 10



MENTAL CALCULATION

ADDITION & SUBTRACTION

COUNTING FORWARDS AND BACKWARDS

Children first meet counting by beginning at one and counting on in ones. Their sense of number is extended by beginning at different numbers and counting forwards and backwards in steps, not only of ones, but also of twos, fives, tens, hundreds, tenths and so on. The image of a number line helps them to appreciate the idea of counting forwards and backwards. They will also learn that, when they add two numbers together, it is generally easier to count on from the larger number rather than the smaller. You will need to review children's 'counting on' strategies, then show them and encourage them to adopt more efficient methods.

	Example Questions	Possible Counting Strategies
YEAR 3	50 + 38	count on in tens then ones from 50
	90 – 27	count back in tens then ones from 90
	34 + 65	count on in tens then ones from 34
	87 – 23	count back in tens then ones from 87
	35 + 15	count on in steps of 5 from 35
YEAR 4	73 – 68	count up from 68, counting 2 to 70 then 3 to 73
	47 + 58	count on 50 from 47, then 3 to 100, then 5 to 105
	124 – 47	count back 40 from 124, then 4 to 80, then 3 to 77
	570 + 300	count on in hundreds from 570
	960 – 500	count back in hundreds from 960
YEAR 5	3.2 + 0.6	count on in tenths
YEAR 6	1.7 + 0.55	count on in tenths and hundredths

ADDITION & SUBTRACTION

REORDERING

Sometimes a calculation can be more easily worked out by changing the order of the numbers. The way in which children rearrange numbers in a particular calculation will depend on which number facts they can recall or derive quickly.

It is important for children to know when numbers can be reordered:

e.g. 2 + 5 + 8 = 8 + 2 + 5 or 15 + 8 – 5 = 15 – 5 + 8 or 23 – 9 – 3 = 23 – 3 – 9

and when they can't be reordered:

e.g. 8 – 5 ≠ 5 – 8

The strategy of changing the order of numbers applies mainly when the question is written down. It is more difficult to reorder numbers if the question is presented orally.

	Example Questions	Possible Counting Strategies
YEAR 3	23 + 54	54 + 23
	12 – 7 – 2	12 – 2 – 7
	13 + 21 + 13	13 + 13 + 21 (using double 13)
YEAR 4	6 + 13 + 4 + 3	6 + 4 + 13 + 3
	17 + 9 – 7	17 – 7 + 9
	28 + 75	75 + 28 (thinking of 28 as 25 + 3)
YEAR 5	12 + 17 + 8 + 3	12 + 8 + 17 + 3
	25 + 36 + 75	25 + 75 + 36
	58 + 47 – 38	58 – 38 + 47
	200 + 567	567 + 200
	1.7 + 2.8 + 0.3	1.7 + 0.3 + 2.8
YEAR 6	3 + 8 + 7 + 6 + 2	3 + 7 + 8 + 2 + 6
	34 + 27 + 46	34 + 46 + 27
	180 + 650	650 + 180 (thinking of 180 as 150 + 30)
	1.7 + 2.8 + 0.3	1.7 + 0.3 + 2.8
	4.7 + 5.6 – 0.7	4.7 – 0.7 + 5.6 = 4 + 5.6

ADDITION & SUBTRACTION

PARTITIONING: COUNTING ON OR BACK

It is important for children to know that numbers can be partitioned into, for example, hundreds, tens and ones, so that 326 = 300 + 20 + 6. In this way, numbers are seen as wholes, rather than as a collection of single digits in columns.

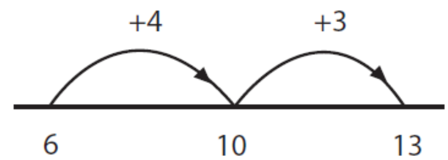
This way of partitioning numbers can be a useful strategy for adding and subtracting pairs of numbers. Both numbers can be partitioned, although it is often helpful to keep the first number as it is and to partition just the second number.

	Example Questions	Possible Counting Strategies
YEAR 3	23 + 45	40 + 5 + 20 + 3 = 40 + 20 + 5 + 3
	68 – 32	60 + 8 – 30 – 2 = 60 – 30 + 8 – 2
YEAR 4	55 + 37	55 + 30 + 7 = 85 + 7
	365 – 40	300 + 60 + 5 – 40 = 300 + 60 – 40 + 5
YEAR 5	43 + 28 + 51	40 + 3 + 20 + 8 + 50 + 1 = 40 + 20 + 50 + 3 + 8 + 1
	5.6 + 3.7	5.6 + 3 + 0.7 = 8.6 + 0.7
	4.7 – 3.5	4.7 – 3 – 0.5
YEAR 6	540 + 280	540 + 200 + 80
	276 – 153	276 – 100 – 50 – 3

ADDITION & SUBTRACTION

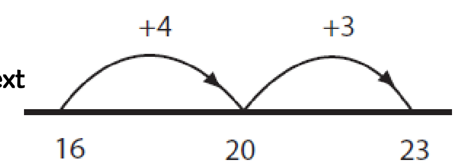
PARTITIONING: BRIDGING THROUGH MULTIPLES OF 10

An important aspect of having an appreciation of number is to know how close a number is to the next or the previous multiple of 10: to recognise, for example, that 47 is 3 away from 50, or that 47 is 7 away from 40.
In mental addition or subtraction, it is often useful to count on or back in two steps, bridging a multiple of 10. The empty number line, with multiples of 10 as 'landmarks', is helpful, since children can visualise jumping to them. For example, 6 + 7 is worked out in two jumps, first to 10, then to 13.
The answer is the last point marked on the line, 13.



Subtraction, the inverse of addition, can be worked out by counting back from the larger number. But it can also be represented as the difference or 'distance' between two numbers. The distance is often found by counting up from the smaller to the larger number, again bridging through multiples of 10 or 100. This method of complementary addition is sometimes called 'shopkeeper's method' because it is like a shop assistant counting out change. So the change from £1 for a purchase of 37p is found by counting coins into the hand: '37p and 3p is 40p, and 10p makes 50p, and 50p makes £1'.
The empty number line can give an image for this method. The calculation 23 – 16 can be built up as an addition:

'16 and 4 is 20, and 3 is 23, so add 4 + 3 for the answer.' In this case the answer of 7 is not a point on the line but is the total distance between the two numbers 16 and 23.
A similar method can be applied to decimals, but here, instead of building up to a multiple of 10, bridging is through the next whole number. So 2.8 + 1.6 is 2.8 + 0.2 + 1.4 = 3 + 1.4.



	Example Questions	**Possible Counting Strategies**
YEAR 3	49 + 32	49 + 1 + 31
	90 – 27	27 + 3 + 60
YEAR 4	57 + 34 or 92 – 25	57 + 3 + 31 or 92 – 2 – 20 – 3
	84 – 35	35 + 5 + 40 + 4
YEAR 5	607 – 288	288 + 12 + 300 + 7
	6070 – 4987	4987 + 13 + 1000 + 70
YEAR 6	1.4 + 1.7 or 5.6 – 3.7	1.4 + 0.6 + 1.1 or 5.6 – 0.6 – 3 – 0.1
	0.8 + 0.35	0.8 + 0.2 + 0.15
	8.3 – 2.8	2.8 + 0.2 + 5.3 or 8.3 – 2.3 – 0.5

MENTAL CALCULATION

ADDITION & SUBTRACTION

PARTITIONING: COMPENSATING

This strategy is useful for adding and subtracting numbers that are close to a multiple of 10, such as numbers that end in 1 or 2, or 8 or 9. The number to be added or subtracted is rounded to a multiple of 10 plus or minus a small number. For example, adding 9 is carried out by adding 10, then subtracting 1; subtracting 18 is carried out by subtracting 20, then adding 2.

A similar strategy works for adding or subtracting decimals that are close to whole numbers. For example: 1.4 + 2.9 = 1.4 + 3 – 0.1 or 2.45 – 1.9 = 2.45 – 2 + 0.1.

	Example Questions	Possible Counting Strategies
YEAR 3	53 + 12	53 + 10 + 2
	53 – 12	53 – 10 – 2
	53 + 18	53 + 20 – 2
	84 – 18	84 – 20 + 2
YEAR 4	38 + 68	38 + 70 – 2
	95 – 78	95 – 80 + 2
	58 + 32	58 + 30 + 2
	64 – 32	64 – 30 – 2
YEAR 5	138 + 69	138 + 70 – 1
	405 – 399	405 – 400 + 1
YEAR 6	$2\frac{1}{2} + 1\frac{3}{4}$	$2\frac{1}{2} + 2 - \frac{1}{4}$
	5.7 + 3.9	5.7 + 4.0 – 0.1
	6.8 – 4.9	6.8 – 5.0 + 0.1

ADDITION & SUBTRACTION

PARTITIONING: USING 'NEAR' DOUBLES

If children have instant recall of doubles, they can use this information when adding two numbers that are very close to each other. So, knowing that 6 + 6 = 12, they can be encouraged to use this to help them find 7 + 6, rather than use a counting on strategy or bridging through 10.

	Example Questions	Possible Counting Strategies
YEAR 3	18 + 16	is double 18 and subtract 2 or double 16 and add 2
	60 + 70	is double 60 and add 10 or double 70 and subtract 1
YEAR 4	76 + 75	is double 76 and subtract 1 or double 75 and add 1
YEAR 5	160 + 170	is double 150, then add 10, then add 20 or double 160 and add 10 or double 170 and subtract 10
YEAR 6	2.5 + 2.6	is double 2.5 and add 0.1 or double 2.6 and subtract 0.1

ADDITION & SUBTRACTION

PARTITIONING: BRIDGING THROUGH 60 TO CALCULATE A TIME INTERVAL

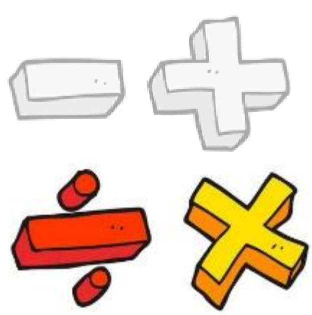
Time is a universal non-metric measure.
A digital clock displaying 9.59 will, in two minutes time, read 10.01 not 9.61. When children use minutes and hours to calculate time intervals, they have to bridge through 60.
So to find the time 20 minutes after 8.50am, for example, children might say 8.50am plus 10 minutes takes us to 9.00am, then add another 10 minutes.

	Examples of Mental Questions
YEAR 3	It is 10.30am. How many minutes to 10.45am?
	It is 3.45pm. How many minutes to 4.15pm?
YEAR 4	I get up 40 minutes after 6.30am. What time is that?
	What is the time 50 minutes before 1.10pm?
	It is 4.25pm. How many minutes to 5.05pm?
YEAR 5	What time will it be 26 minutes after 3.30am?
	What was the time 33 minutes before 2.15pm?
	It is 4.18pm. How many minutes to 5.00pm? 5.26pm?
YEAR 6	It is 08.35. How many minutes is it to 09.15?
	It is 11.45. How many hours and minutes is it to 15.20?
	A train leaves London for Leeds at 22.33. The journey takes 2 hours 47 minutes. What time does the train arrive?

MENTAL CALCULATION

MULTIPLICATION AND DIVISION

- Knowing multiplication and division facts to 12 x 12
- Doubling and halving
- Multiplying and dividing by multiples of 10
- Multiplying and dividing by single-digit numbers and multiplying and dividing by two-digit numbers
- Finding fractions, decimals and percentages



MULTIPLICATION AND DIVISION

MULTIPLICATION AND DIVISION FACTS TO 12x12

Fluent recall of multiplication and division facts relies on regular opportunities for practice. Generally, frequent short sessions are more effective than longer, less frequent sessions. It is crucial that the practice involves as wide a variety of activities, situations, questions and language as possible and that it leads to deriving and recognising number properties, such as doubles and halves, odd and even numbers, multiples, factors and primes.

	Example Questions
YEAR 3	Derive and recall doubles of multiples of 10 to 100 and corresponding halves
	Derive and recall multiplication facts for the 2, 3, 4, 5, 6 and 10 times-tables and corresponding division facts
	Recognise multiples of 2, 3, 4, 5, 6 and 10 up to the tenth multiple
YEAR 4	Identify doubles of two-digit numbers and corresponding halves
	Derive doubles of multiples of 10 and 100 and corresponding halves
	Derive and recall multiplication facts up to 10 × 10 and corresponding division facts
	Recognise multiples of 2, 3, 4, 5, 6, 7, 8, 9 and 10 up to the tenth multiple
YEAR 5	Recall squares of numbers to 10 × 10
	Use multiplication facts to derive products of pairs of multiples of 10 and 100 and corresponding division facts
YEAR 6	Recall squares of numbers to 12 × 12 and derive corresponding squares of multiples of 10
	Use place value and multiplication facts to derive related multiplication and division facts involving decimals (e.g. 0.8 × 7, 4.8 ÷ 6)
	Identify factor pairs of two-digit numbers
	Identify prime numbers less than 100

MENTAL CALCULATION

MULTIPLICATION AND DIVISION

DOUBLING AND HALVING

The ability to double numbers is useful for multiplication.
Historically, multiplication was carried out by a process of doubling and adding. Most people find doubles the easiest multiplication facts to remember, and they can be used to simplify other calculations.
Sometimes it can be helpful to halve one of the numbers in a multiplication calculation and double the other.

	Example Questions
YEAR 3	Double multiples of 10 to 100, e.g. double 90, and corresponding halves
	Double multiples of 5 to 100 and find the corresponding halves, e.g. double 85, halve 170
YEAR 4	Double any two-digit number and find the corresponding halves, e.g. double 47, half of 94
	Double multiples of 10 and 100 and find the corresponding halves, e.g. double 800, double 340, half of 1600, half of 680
YEAR 5	Form equivalent calculations and use doubling and halving, e.g. • multiply by 4 by doubling twice, e.g. 16 × 4 = 32 × 2 = 64 • multiply by 8 by doubling three times, e.g. 12 × 8 = 24 × 4 = 48 × 2 = 96 • divide by 4 by halving twice, e.g. 104 ÷ 4 = 52 ÷ 2 = 26 • divide by 8 by halving three times, e.g. 104 ÷ 8 = 52 ÷ 4 = 26 ÷ 2 = 13 • multiply by 5 by multiplying by 10 then halving, e.g. 18 × 5 = 180 ÷ 2 = 90 • multiply by 20 by doubling then multiplying by 10, e.g. 53 × 20 = 106 × 10 = 1060
	Multiply by 50 by multiplying by 100 and halving
	Multiply by 25 by multiplying by 100 and halving twice
YEAR 6	Double decimals with units and tenths, e.g. double 7.6, and find the corresponding halves, e.g. half of 15.2 Form equivalent calculations and use doubling and halving, e.g. • divide by 25 by dividing by 100 then multiplying by 4 e.g. 460 ÷ 25 = 4.6 × 4 = 18.4 • divide by 50 by dividing by 100 then doubling e.g. 270 ÷ 50 = 2.7 × 2 = 5.4

MULTIPLYING AND DIVIDING BY MULTIPLES OF 10

Being able to multiply by 10 and multiples of 10 depends on an understanding of place value and knowledge of multiplication and division facts. This ability is fundamental to being able to multiply and divide larger numbers.

	Expectations with Example
YEAR 3	Multiply one-digit and two-digit numbers by 10 or 100, e.g. 7 × 100, 46 × 10, 54 × 100
	Change pounds to pence, e.g. £6 to 600 pence, £1.50 to 150 pence
YEAR 4	Multiply numbers to 1000 by 10 and then 100, e.g. 325 × 10, 42 × 100
	Divide numbers to 1000 by 10 and then 100 (whole-number answers), e.g. 120 ÷ 10, 600 ÷ 100, 850 ÷ 10
	Multiply a multiple of 10 to 100 by a single-digit number, e.g. 60 × 3, 50 × 7
	Change hours to minutes; convert between units involving multiples of 10 and 100, e.g. centimetres and millimetres, centilitres and millilitres, and convert between pounds and pence, metres and centimetres, e.g. 599 pence to £5.99, 2.5m to 250cm
YEAR 5	Multiply and divide whole numbers and decimals by 10, 100 or 1000, e.g. 4.3 × 10, 0.75 × 100, 25 ÷ 10, 673 ÷ 100
	Divide a multiple of 10 by a single-digit number (whole number answers), e.g. 80 ÷ 4, 270 ÷ 3
	Multiply pairs of multiples of 10, and a multiple of 100 by a single digit number, e.g. 60 × 30, 900 × 8
	Multiply by 25 or 50, e.g. 48 × 25, 32 × 50 using equivalent calculations, e.g. 48 x 100 ÷ 4, 32 x 100 ÷ 2
	Convert larger to smaller units of measurement using decimals to one place, e.g. change 2.6 kg to 2600 g, 3.5 cm to 35 mm, and 1.2 m to 120 cm
YEAR 6	Multiply pairs of multiples of 10 and 100, e.g. 50 × 30, 600 × 20
	Divide multiples of 100 by a multiple of 10 or 100 (whole number answers), e.g. 600 ÷ 20, 800 ÷ 400, 2100 ÷ 300
	Divide by 25 or 50
	Convert between units of measurement using decimals to two places, e.g. change 2.75 l to 2750 ml, or vice versa

MULTIPLICATION AND DIVISION

MULTIPLYING AND DIVIDING BY SINGLE-DIGIT NUMBERS AND MULTIPLYING BY TWO-DIGIT NUMBERS

Once children are familiar with some multiplication facts, they can extend their skills.

- One strategy is to partition one of the numbers and use the distributive law of multiplication over addition. So, for example, 6 × 7 = 6 × (5 + 2) = 6 × 5 + 6 × 2 or, in words, 'seven sixes are five sixes plus two sixes'. Subtraction can be used similarly, so 'nine eights are ten eights minus one eight'.
- Another strategy is to make use of factors, so 7 × 6 is seen as 7 × 3 × 2.

Once children understand the effect of multiplying and dividing by 10, they can start to extend their multiplication and division skills to larger numbers.

- A product such as 26 × 3 can be worked out by partitioning 26 into 20 + 6, multiplying each part by 3, then recombining.
- One strategy for multiplication by 2, 4, 8, 16, 32, ... is to use doubling, so that 9 × 8 is seen as 9 × 2 × 2 × 2. A strategy for dividing by the same numbers is to use halving.
- A strategy for multiplying by 50 is to multiply by 100, then halve, and for multiplying by 25 is to multiply by 100 then divide by 4.

Since each of these strategies involves at least two steps, most children will find it helpful to make jottings of the intermediate steps in their calculations.

	Example Questions
YEAR 4	Find one quarter by halving one half
	Multiply numbers to 20 by a single-digit number, e.g. 17 × 3
YEAR 5	Multiply and divide two-digit numbers by 4 or 8, e.g. 26 × 4, 96 ÷ 8
	Multiply two-digit numbers by 5 or 20, e.g. 32 × 5, 14 × 20
	Multiply by 25 or 50, e.g. 48 × 25, 32 × 50
YEAR 6	Multiply a two-digit and a single-digit number, e.g. 28 × 7
	Divide a two-digit number by a single-digit number e.g. 68 ÷ 4
	Divide by 25 or 50, e.g. 480 × 25, 3200 × 50
	Find new facts from given facts, e.g. • given that three oranges cost 24p, find the cost of four oranges

MENTAL CALCULATION

MULTIPLICATION AND DIVISION

FRACTIONS, DECIMALS AND PERCENTAGES

Children need an understanding of how fractions, decimals and percentages relate to each other. For example, if they know that 12, 0.5 and 50% are all ways of representing the same part of a whole, then they can see that the calculations:

half of 40 ½ × 40 40 × ½ 40 × 0.5 0.5 × 40 50% of 40

are different versions of the same calculation. Sometimes it might be easier to work with fractions, sometimes with decimals and sometimes with percentages.

There are strong links between this section and the earlier section 'Multiplying and dividing by multiples of 10'.

	Example Questions
YEAR 3	Find half of any multiple of 10 up to 200, e.g. halve 170
	Find $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$ and $\frac{1}{10}$ of numbers in the 2, 3, 4, 5 and 10 times tables
YEAR 4	Find half of any even number to 200
	Find unit fractions and simple non-unit fractions of whole numbers or quantities, e.g. $\frac{3}{8}$ of 24
	Recall fraction and decimal equivalents for one-half, quarters, tenths and hundredths, e.g. recall the equivalence of 0.3 and $\frac{3}{10}$, and 0.03 and $\frac{3}{100}$
YEAR 5	Recall percentage equivalents of one-half, one-quarter, three-quarters, tenths and hundredths
	Find fractions of whole numbers or quantities, e.g. $\frac{2}{3}$ of 27, $\frac{4}{5}$ of 70 kg
	Find 50%, 25% or 10% of whole numbers or quantities, e.g. 25% of 20 kg, 10% of £80
YEAR 6	Recall equivalent fractions, decimals and percentages for hundredths, e.g. 35% is equivalent to 0.35 or $\frac{35}{100}$
	Find half of decimals with units and tenths, e.g. half of 3.2
	Find 10% or multiples of 10%, of whole numbers and quantities, e.g. 30% of 50 ml, 40% of £30, 70% of 200 g

Kilburn Junior School

Written Calculation Guidance

EMBARK
FEDERATION

Written Arithmetic

A solid grounding in number- both the concept of place value and the ability to calculate in all four operations for whole numbers and decimals- are essential to understanding mathematics and, indeed, are essential for coping with everyday life with confidence and independence. At Kilburn Junior school, we teach children a range of calculation strategies to support their mathematical reasoning and problem solving. We ensure children have regular practice in these methods so that they become proficient and confident mathematicians.

Addition

8938 + 357 = ?

	8	9	3	8
+		3	5	7
	9	2	9	5
	1		1	

Children should be able to add whole numbers very quickly and efficiently up to at least ten million by the end of year 6.

5789 + 6832 + 2764 = ?

	5	7	8	9
	6	8	3	2
+	2	7	6	4
1	5	3	8	5
1	2	1	1	

Children should also be proficient in using the formal written methods of addition to add three (or more) whole numbers or decimals together.

893.8 + 35.7 = ?

	8	9	3 •	8
+		3	5 •	7
	9	2	9 •	5
	1		1	

Children should be proficient in adding decimals using the same strategy as for whole numbers but inserting decimal points into their working out.

245.9 + 84.66 = ?

2	4	5 •	9	0
+	8	4 •	6	6
3	3	0 •	5	6
1	1	1		

For numbers with different amounts of decimal places, children should recognise the need to align the digits by place value in their working out (including inserting place holders) to support working out.

$\frac{1}{5} + \frac{3}{5} = ?$

$\frac{1}{5}$	+	$\frac{3}{5}$	=	$\frac{4}{5}$

Children should be able to add fractions with the same denominator recognising that only the numerators will change in their answer.

$\frac{2}{6} + \frac{3}{18} = ?$

$\frac{2}{6}$	+	$\frac{3}{18}$	=	?
↓				
$\frac{6}{18}$	+	$\frac{3}{18}$	=	$\frac{9}{18}$

To add unlike fractions, children need to convert the fractions to a common denominator. In this case, the sixths can be converted to eighteenths by multiplying both digits by three.

$\frac{1}{4} + \frac{1}{3} = ?$

$\frac{1}{4}$	+	$\frac{1}{3}$	=	?
↓		↓		
$\frac{3}{12}$	+	$\frac{4}{12}$	=	$\frac{7}{12}$

When one denominator isn't a factor of the other, both fractions need to be converted to a common denominator. Both 4 and 3 share 12 as a multiple.

Subtraction

7436 - 314 = ?

	7	4	3	6
-		3	1	4
	7	1	2	2

Children need to be careful to ensure that the bottom number is always subtracted *from* the top number and not vice versa.

8938 - 357 = ?

	8	$^{8}\cancel{9}$	$^{1}3$	8
-		3	5	7
	8	5	8	1

Where the digits in the bottom number are larger than those in the top, children exchange from the next column to the left.

464.8 – 235.6 = ?

	4	$^{5}\cancel{6}$	$^{1}4$ •	8
-	2	3	5 •	6
	2	2	9 •	2

Children should be proficient in subtracting decimals using the same strategy as for whole numbers but inserting decimal points into their working out.

467.7 - 37.36 = ?

4	6	7 •	$^{6}\cancel{7}$	$^{1}0$
-	3	7 •	3	6
4	3	0 •	3	4

As with addition, for numbers with different amounts of decimal places, children should recognise the need to align the digits by place value in their working out (including inserting place holders) to support working out.

$$\frac{5}{9} - \frac{3}{9} = ?$$

$\frac{5}{9}$	-	$\frac{3}{9}$	=	$\frac{2}{9}$

As with addition, children should recognise that only the numerators will change in their answer where the denominators are the same.

$$\frac{7}{10} - \frac{2}{5} = ?$$

$\frac{7}{10}$	-	$\frac{2}{5}$	=	?
		↓		
$\frac{7}{10}$	-	$\frac{4}{10}$	=	$\frac{3}{10}$

To subtract unlike fractions, children need to convert the fractions to a common denominator. One or both fractions may need to be converted to carry out the calculation.

$$1\frac{1}{4} - \frac{3}{4} = ?$$

$1\frac{1}{4}$	-	$\frac{3}{4}$	=	?
↓				
$\frac{5}{4}$	-	$\frac{3}{4}$	=	$\frac{2}{4}$

To subtract a mixed number where the first fraction has a smaller numerator than the second, children need to recognise that they must convert the mixed number to an improper fraction to successfully calculate the answer.

Multiplication

57 x 3 = ?

		5	7
x			3
	1	7	1
	1	2	

627 x 4 = ?

		6	2	7
x				4
	2	5	0	8
		1	2	

364.7 x 7 = ?

	3	6	4 •	7
x				7
2	5	5	2 •	9
	4	3	4	

To enable children to multiply whole numbers or decimals by another one-digit number, the above method can used. This methods partitions the numbers by their place value and allows children to apply their automatic recall of the multiplication tables up to 12x12 to quickly calculate. This method also adds the separate calculations together as the children complete them. Including a decimal in the answer in line with the multiplier doesn't change the process at all.

27 x 43 = ?

			2	7
x			4	3
			$_{2}$8	1
	1	$_{2}$0	8	0
	1	1	6	1
		1		

627 x 32= ?

		6	2	7
x			3	2
	1	2	$_{1}$5	4
1	8	$_{2}$8	1	0
2	0	0	6	4
1	1			

3627 x 54 = ?

		3	6	2	7
	x			5	4
	1	$_{2}$4	$_{1}$5	$_{2}$0	8
1	$_{3}$8	$_{1}$1	$_{3}$3	5	0
1	9	5	8	5	8

'Long multiplication' is essentially the same as the method above but the two-digit multiplier is also partitioned and each calculation is carried out separately on two lines before being recombined through adding the two answers together at the bottom of the calculation. The multiplication for the ones digit is carried out on the top line of the answer, the tens digit on the second line of the answer (with a place holder to increase the different answers' place value by 10) and the combined answer is shown in the final line.

$\frac{2}{5} x \frac{3}{4} = ?$

2 x 3 = 6 →				
$\frac{2}{5}$	x	$\frac{3}{4}$	=	$\frac{6}{20}$
5 x 4 = 20 →				

$\frac{5}{6} x \frac{3}{7} = ?$

5 x 3 = 15 →				
$\frac{5}{6}$	x	$\frac{3}{7}$	=	$\frac{15}{42}$
6 x 7 = 42 →				

$3\ x \frac{1}{5} = ?$

3 x 1 = 6 →				
$\frac{3}{1}$	x	$\frac{1}{5}$	=	$\frac{3}{5}$
1 x 5 = 20 →				

To multiply fractions, simply multiply the numerators together and multiply the denominators together to find the answer. Simple! Children only need to simplify fractions if this is specified in the question. To multiply fractions by a whole number, convert the whole number into a fraction using 1 as the denominator.

Division

738 ÷ 3 = ?

	2	4	6
3	7	^{1}3	^{1}8

643 ÷ 5 = ?

	1	2	8	r 3
5	6	^{1}4	^{4}3	

9322 ÷ 4 = ?

	2	3	3	0 •	5
4	9	^{1}3	^{1}2	2 •	^{2}0

Children need to be proficient in dividing whole numbers by a one-digit divisor where the answer is a whole number, or for questions where the answer is not a whole number. The context of the question they are finding a solution to will dictate if they need to only express the answer as a remainder or be more specific and find the solution in the form of a decimal.

For 'long division', children are working with times tables over the 12x12 boundary that children are taught to learn by heart. We, therefore, encourage children to write down the first few multiples of the divisor to support them in subtracting different groups of the divisor away from the dividend to find the total of the division calculation.

644 ÷ 14= ?

14	28	42	56	70	84
			4	6	
1	4	6	4	4	
	-	5	6	↓	
			8	4	
			8	4	
				0	

4956 ÷ 21= ?

21	42	63	84	105	126
			2	3	6
2	1	4	9	5	6
	-	4	2	↓	↓
			7	5	
		-	6	3	
			1	2	6
		-	1	2	6
					0

$\frac{5}{6} \div \frac{2}{7} = ?$

Keep ↓		Flip ↓		
$\frac{5}{6}$	x	$\frac{7}{2}$	=	$\frac{35}{12}$
	↑ Change		or	$1\frac{11}{12}$

$\frac{8}{9} \div 5 = ?$

Keep ↓		Flip ↓		
$\frac{8}{9}$	x	$\frac{1}{5}$	=	$\frac{8}{45}$
	↑ Change			

$3 \div \frac{2}{5} = ?$

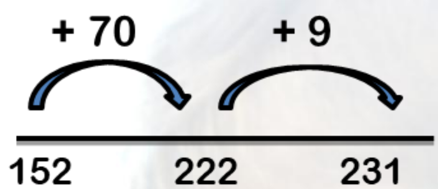
Keep ↓		Flip ↓		
$\frac{3}{1}$	x	$\frac{2}{5}$	=	$\frac{6}{5}$
	↑ Change		or	$1\frac{1}{5}$

We use the mnemonic **KFC (Keep, Flip, Change)** to teach children how to divide with fractions. The first number is **kept** (if a whole number like 3, we turn it into $\frac{3}{1}$) , the second number is **flipped** (if a whole is the divisor, it is flipped from $\frac{3}{1}$ to become $\frac{1}{3}$) and the division symbol is **changed** to a multiplication. Children only need to simplify or convert fractions from improper to mixed numbers if this is specified in the question.

Additional Teaching Aids for Written Arithmetic

At all levels, number lines can be used by children to support arithmetic. Even with strong formal, vertical methods, numbers lines can be used for mathematical proof, demonstrating conceptual understanding or modular arithmetic such as working with time.

$$152 + 79 = 231$$



Variations:

- Counting on in ones
- Counting on and bridging through multiples
- Counting on after partitioning addend
- Bridging through numbers other than multiples of ten

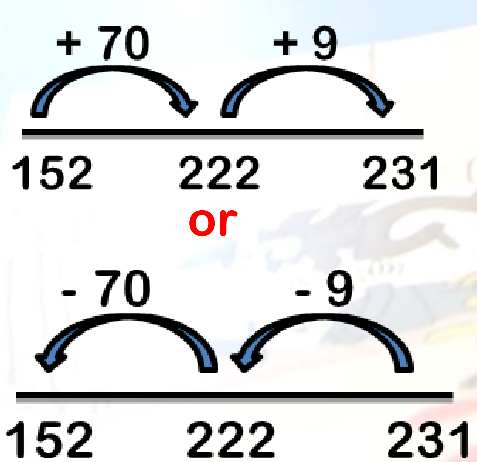
In order to demonstrate place value of each separate calculation in the vertical method, an expanded method can be used. This allows for easier validation of answers for pupils and monitoring by staff.

$$\begin{array}{r l} 1\ 4\ 7 & \\ +\ 2\ 4\ 6 & \\ \hline 1\ 3 & (7+6) \\ 8\ 0 & (40+40) \\ 3\ 0\ 0 & (100+200) \\ \hline 3\ 9\ 3 & \end{array}$$

Variations:

- Also used for decimals where appropriate
- Notation at the side of the answers optional but demonstrate understanding of place value initially

As with addition, number lines can be used with subtraction for mathematical proof, demonstrating conceptual understanding or modular arithmetic. Counting up or counting down can be used based on preference or on the numbers that present in the calculation.



Variations:

- Counting on or back in ones
- Counting on or back and bridging through multiples
- Counting on or back after partitioning
- Bridging through numbers other than multiples of ten

Additional Teaching Aids for Written Arithmetic

Children will need strong knowledge of times tables facts to support this method. Addition method may need to used to recombine the answers from the partitioned multiplier / multiplicand.

x	40	2
7	280	14

x	200	60	2
6	1200	360	12

x	60	2
40	2400	80
6	360	12

Expanded method relies on strong mental multiplication of units and multiples of 10 or two multiples of 10… etc. Again, writing calculations next to the individual multiplications can aid validation of answers.

```
      6 2
 x    4 6
 --------
      1 2   (2x6)
    3 6 0   (60x6)
      8 0   (2x40)
  2 4 0 0   (60x40)
 --------
  2 8 5 2
  1
```

Variations:

- TU x U
- HTU x U
- TU x TU
- HTU x TU
- ThHTU - HTU
- Etc…

As with subtraction, number lines can be used to demonstrate that division is the same as counting back in groups of the divisor and keeping track of how many groups 'fit into' the dividend. This can be done individually or in 'chunks' for both whole number answers and remainders.

54 ÷ 6 = 9

-6 -6 -6 -6 -6 -6 -6 -6 -6

0 6 12 18 24 30 36 42 48 54

or

93 ÷ 4 = 9 r 1

3 x 4 = 12   20 x 4 = 80

0 1 12 93

APPENDIX 1

(Basic Terminology of the Four Operations)

Addend		Addend		Sum
21	+	34	=	55

Minuend		Subtrahend		Difference
64	-	22	=	42

Multiplicand		Multiplier		Product
3	x	7	=	21

Dividend		Divisor		Quotient
45	÷	9	=	5