

## Stages in Addition

### Addition - Early Stages (EYFS)

Children will engage in a wide variety of songs and rhymes, games and activities. They will begin to relate addition to **combining two groups of objects**, first by **counting all** and then by **counting on** from the largest number.

They will find one more than a given number.

In practical activities and through discussion they will begin to use the vocabulary involved in addition.



'You have five apples and I have three apples. How many apples altogether?'

### Addition - Year One

- **Given a number, identify one more**
- **Read, write and interpret mathematical statements involving addition (+) and the equals (=) sign**
- **Add one- digit and two-digit numbers within 20, including zero**
- **Solve missing number problems eg  $10 + \square = 16$**

**NB** Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Children will continue to practise counting on from any number e.g. 'Put five in your head and count on four.'

Initially use a **number track** to count on for addition, counting on from the largest number:

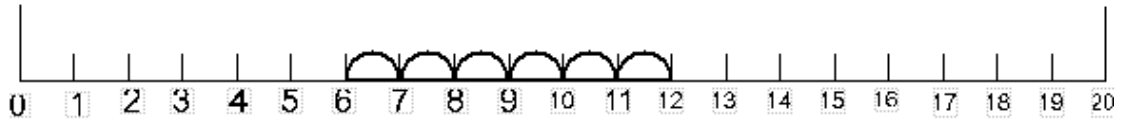


$$5 + 4 = 9$$

'Put your finger on number five. Count on (count forwards) four.'

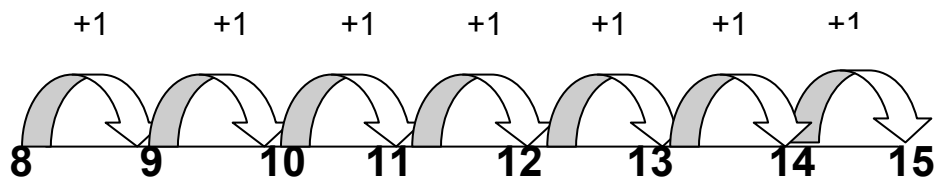
Then progress to a **marked number line**:

$$6 + 6 = 12$$



'Put your finger on number six and count on six.'

$$8 + 7 = 15$$
 'Put your finger on number eight and count on seven.'



Ensure children are confident with using a marked number line before moving on to an empty number line (see year two guidance).

Continue to practise counting on from the largest number for addition with totals within 20.

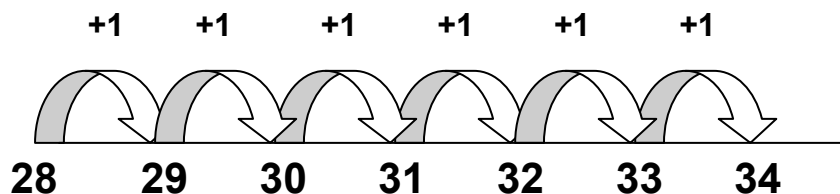
## Addition - Year Two

- Add numbers using concrete objects, pictorial representations, and mentally, including:
  - A two digit number and ones
  - A two digit number and tens
  - Two two-digit numbers
  - Three one-digit numbers (see mental maths policy)

**NB** Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

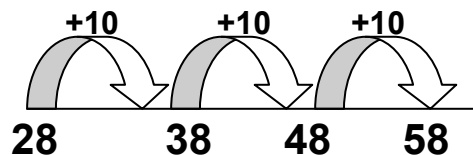
Counting on in ones using an **empty number line**, within 100...

$$28 + 6 = 34$$



...and in tens

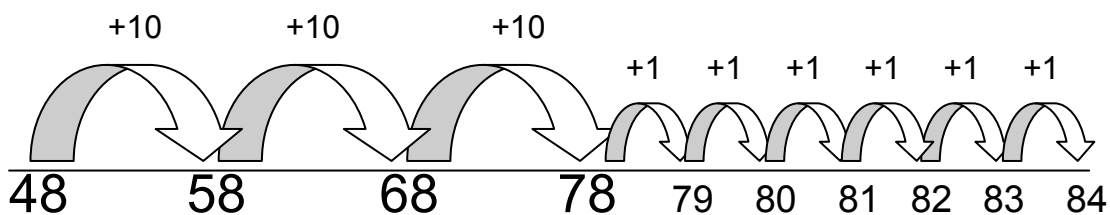
$$28 + 30 = 58$$



Use in conjunction with a **100 square** to show jumps of tens.

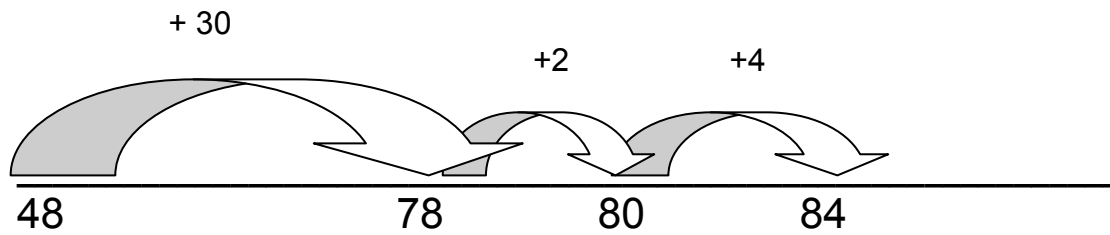
$$48 + 36 = 84$$

'Put the biggest number first (48), and then partition the smaller number (36 = 30 + 6) and count on: 48 + 30 + 6.'



Use in conjunction with a **100 square** to show jumps of tens and ones.

If children are confident, use more efficient jumps...



Use in conjunction with a **100 square** to show jumps of tens and ones/units.

Also use the **partitioning method** to add two two-digit numbers:

$$\begin{array}{r} 43 + 25 = 68 \\ \begin{array}{l} / \quad \backslash \\ 40 \quad 3 \end{array} \quad \begin{array}{l} / \quad \backslash \\ 20 \quad 5 \end{array} \end{array}$$
$$\begin{array}{l} 40 + 20 = 60 \\ 3 + 5 = 8 \\ 60 + 8 = 68 \end{array}$$

'Partition the numbers into tens and ones/units.  
Add the tens together and then add the ones/units together.  
Recombine to give the answer'.

Then move on to calculations that **bridge** the tens:

$$48 + 36 = 40 + 8 + 30 + 6$$
$$\begin{array}{l} 40 + 30 = 70 \\ 8 + 6 = 14 \\ 70 + 14 = 84 \\ 48 + 36 = 84 \end{array}$$

This is an alternative way of recording the partitioning method.

Further develop addition with numbers that bridge 100, using a **200 grid** to support.

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

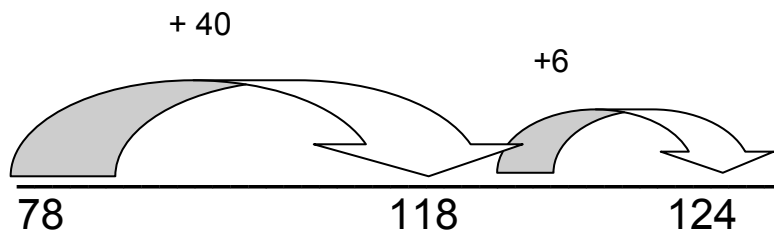
## Addition - Year Three

- Add numbers with up to three digits, using formal written method of columnar addition

**NB** Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Further develop the use of the **empty number line** with calculations that **bridge 100**:

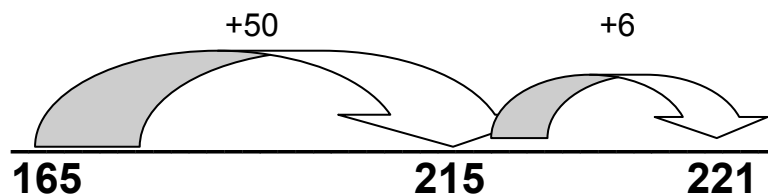
$$78 + 46 = 124$$



Use a **200 grid** to support counting on in tens and bridging 100...

... and with addition of a three-digit and a two-digit number:

$$165 + 56 = 221$$



Further develop the **partitioning method** with calculations that **bridge 100**:

$$85 + 37 = 80 + 5 + 30 + 7$$

$$80 + 30 = 110$$

$$5 + 7 = 12$$

$$110 + 12 = 122$$

$$85 + 37 = 122$$

The partitioning method can also be used with three-digit numbers.

Introduce the **expanded written method** with the calculation presented both horizontally and vertically (in columns).

Initially use calculations where it has not been necessary to bridge across the tens or hundreds:

$$63 + 32 = 95$$

$$\begin{array}{r} 60 + 3 \\ + \quad 30 + 2 \\ \hline 90 + 5 = 95 \end{array}$$

'Partition the numbers into tens and ones/units. Add the tens together and then add the ones/units together. Recombine to give the answer.'

Then...

$$\begin{array}{r} \phantom{+} 63 \\ + \phantom{+} 32 \\ \hline + \phantom{+} 5 \quad (3 + 2) \\ \hline \phantom{+} 90 \quad (60 + 30) \\ \hline \phantom{+} 95 \end{array}$$

Add the least significant digits (units) together first and then the tens in preparation for the formal written method.

This will lead into the **formal written method**...

$$\begin{array}{r} 63 \\ + 32 \\ \hline 95 \end{array}$$

Use the language of place value to ensure understanding: 'Three add two equals five. Write five in the units column. 60 add 30 equals 90. Write 9 (90) in the tens column.'

**NB** Informal/mental methods would be more appropriate for numbers of this size, but use two-digit numbers when introducing the columnar method.

Then introduce calculations where it is necessary to bridge, returning to an **expanded method** initially:

$$68 + 24 = 92$$

$$\begin{array}{r} 60 + 8 \\ + \quad 20 + 4 \\ \hline 80 + 12 = 92 \end{array}$$

'Partition the numbers into tens and ones/units. Add the tens together and then add the ones/units together. Recombine to give the answer.'

Then...

$$\begin{array}{r} 68 \\ + 24 \\ \hline 12 \quad (8 + 4) \\ + 80 \quad (60 + 20) \\ \hline 92 \end{array}$$

Add the least significant digits (units) together first and then the tens in preparation for the formal written method.

**If children are ready**, introduce the **formal written method**, where it is necessary to 'carry' ten from the units to the tens column:

$$\begin{array}{r} 68 \\ + 24 \\ \hline 92 \\ \hline 1 \end{array}$$

Use the language of place value to ensure understanding: 'Eight add four equals 12. Write two in the units column and 'carry' one (10) across into the tens column. 60 add 20 and the ten that we 'carried' equals 90. Write 9 (90) in the tens column. 92 is the answer.

The digit that has been 'carried' should be recorded under the line in the correct column.

**When children are confident**, extend with examples where it is necessary to bridge across the tens and the hundreds:

$$76 + 47 = 123$$

$$\begin{array}{r} 70 + 6 \\ + 40 + 7 \\ \hline 110 + 13 = 123 \end{array}$$

'Partition the numbers into tens and ones/units. Add the tens together and then add the ones/units together. Recombine to give the answer.'

Then...

$$\begin{array}{r} 76 \\ + 47 \\ \hline 13 \quad (7 + 6) \\ + 110 \quad (70 + 40) \\ \hline 123 \end{array}$$

Add the least significant digits (units) together first and then the tens in preparation for the formal written method.

If children are ready introduce the **formal written method**, where it is necessary to 'carry' across the columns and bridge 100:

$$76 + 47 = 123$$

$$\begin{array}{r} 47 \\ + 76 \\ \hline 123 \\ \hline \end{array}$$

Use the language of place value to ensure understanding: 'Seven add six equals 13. Write three in the units column and 'carry' one (10) across into the tens column. 40 add 70 and the ten that we 'carried' equals 120. Write 2 (20) in the tens column and 'carry' one (100) across into the hundreds column (100).

The digits that have been 'carried' should be recorded under the line in the correct column.

**If children are confident**, further develop with the addition of a three- digit number and a two -digit number:

$$178 + 43 = 221$$

$$\begin{array}{r} 178 \\ + 43 \\ \hline 221 \\ \hline \end{array}$$

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

## Addition - Year Four

- Add numbers with up to 4 digits using the formal written method of columnar addition where appropriate

**NB** Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to teach the use of **empty number lines** with three and four digit numbers, as appropriate.

Further develop the formal written method of addition, with three-digit numbers. Revisit the **expanded method** first, if necessary:

$$176 + 147 = 323$$

$$\begin{array}{r} 176 \\ + 147 \\ \hline + 13 \quad (7 + 6) \\ 110 \quad (70 + 40) \\ \hline 200 \quad (100 + 100) \\ \hline 323 \end{array}$$

This will lead into the **formal written method**...

$$176 + 147 = 323$$

$$\begin{array}{r} 147 \\ + 176 \\ \hline 323 \\ \hline \underset{1}{1} \end{array}$$

Use the language of place value to ensure understanding:

'Seven add six equals 13. Write three in the units column and 'carry' one across into the tens column (10). 40 add 70 and the ten that we carried equals 120. Write 2 in the tens column (20) and 'carry' 1 across into the hundreds column (100). 100 add 100 and the 100 that has been carried equals 300. Write 3 in the hundreds column (300).

The digits that have been 'carried' should be recorded under the line in the correct column.

If children are confident, introduce the addition of a four-digit number and a three digit number:

$$1845 + 526 = 2371$$

$$\begin{array}{r} 1845 \\ + 526 \\ \hline 2371 \\ \hline \end{array}$$

Continue to develop with addition of two four-digit numbers and with decimals (in the context of money or measures).

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

## Addition - Year Five

- **Add whole numbers with more than 4 digits, including using formal written method (columnar addition)**

**NB** Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to teach the use of **empty number lines** with larger numbers (and decimals), as appropriate.

Continue to develop the **formal written method for addition** with larger numbers (and decimal numbers) and with the addition of three or more numbers:

$$21848 + 1523 = 23371$$

$$\begin{array}{r} 21848 \\ + 1523 \\ \hline 23371 \\ \small 1 \quad 1 \end{array}$$

Continue to use the language of place value to ensure understanding. Ensure that the digits that have been 'carried' are recorded under the line in the correct column.

Use the **formal written method** for the addition of decimal numbers:

$$£154.75 + £233.82 = £388.57$$

$$\begin{array}{r} 154.75 \\ + 233.82 \\ \hline 388.57 \\ \small 1 \end{array}$$

Continue to use the language of place value to ensure understanding.

Ensure that the decimal points line up.

Continue to practise and apply the formal written method throughout Y5.

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

## Addition - Year Six

No objectives have been included in the programmes of study explicitly related to written methods for addition in Y6. However, there is an expectation that children will continue to practise and use the **formal written method for larger numbers and decimals** and use these methods when solving problems, when appropriate (see previous year's guidance for methods).

Our aim is that by the end of Y6, children **use mental methods (with jottings)** when appropriate, but for calculations that they cannot do in their heads, they use an efficient **formal written method** accurately and with confidence.

## Stages in Subtraction

### Subtraction - Early Stages (EYFS)

Children will engage in a variety of counting songs and rhymes and practical activities.

In practical activities and through discussion they will begin to use the vocabulary associated with subtraction.

They will find one less than a given number.

They will begin to relate subtraction to 'taking away' **using objects** to count 'how many are left' after some have been taken away.

$$6 - 2 = 4$$



'Take two apples away. How many are left?'

Children will begin to count back from a given number.

### Subtraction - Year One

- **Given a number, identify one less**
- **Read, write and interpret mathematical statements involving subtraction (-) and the equals (=) sign**
- **Subtract one- digit and two-digit numbers within 20, including zero**
- **Solve missing number problems eg  $20 - \square = 15$**

**NB** Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Children will continue to practise counting back from a given number.

Initially use a **number track** to **count back** for subtraction:

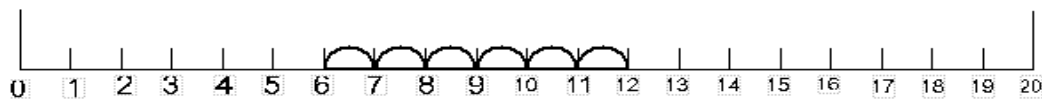


$$9 - 5 = 4$$

'Put your finger on number nine. Count back five.'

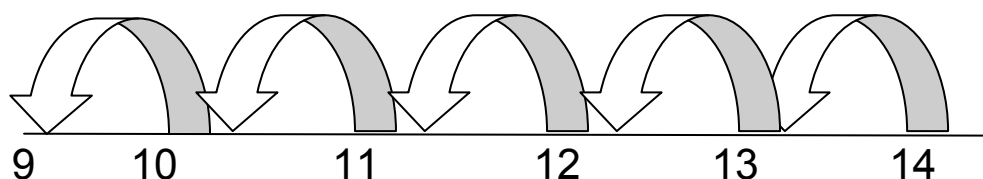
Then progress to a **marked number line**:

$$12 - 6 = 6$$



'Put your finger on number twelve and count back six.'

$$14 - 5 = 9$$



'Put your finger on number 14 and count back five.'

**NB** Ensure children are confident with using a **marked number line** before moving on to an empty number line (see year two guidance).

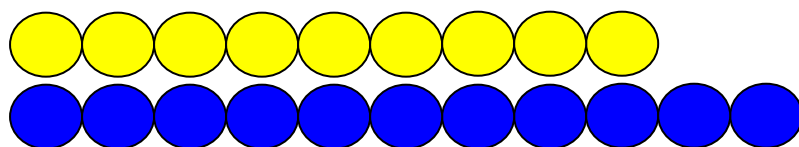
Continue to practise counting back for subtraction with numbers within 20.

### Counting on to find a small difference:

Introduce complementary addition to find differences (only use for **small** differences). The use of models is extremely important here to understand the idea of "difference".

**Count up** from the smallest number to the largest to **find the difference** using resources, e.g. cubes, beads, number tracks/lines:

$$11 - 9 = 2$$



The **difference between** nine and eleven is two.

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

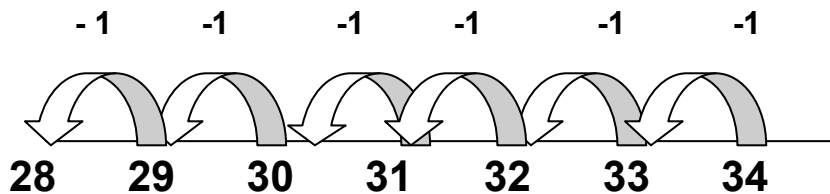
## Subtraction - Year Two

- Subtract numbers using concrete objects, pictorial representations, and mentally, including:
  - A two digit number and ones
  - A two digit number and tens
  - Two two-digit numbers

**NB** Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

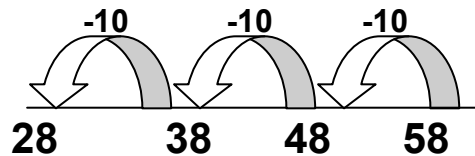
Counting back using an **empty number line** within 100, in ones...

$$34 - 6 = 28$$



...and in tens:

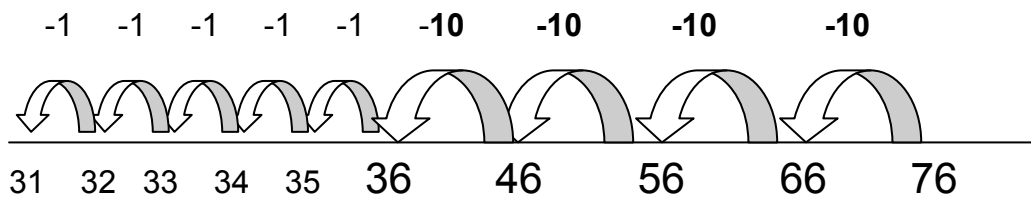
$$58 - 30 = 28$$



Use in conjunction with a **100 square** to show jumps of tens.

**Subtraction, using partitioning, on an empty number line:**

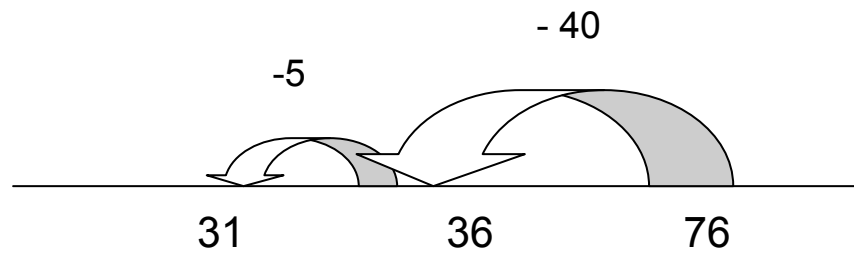
$$76 - 45 = 31$$



Use in conjunction with a **100 square** to show jumps of tens and ones.

If children are confident, use more efficient jumps:

$$76 - 45 = 31$$



$$76 - 40 - 5 = 31$$

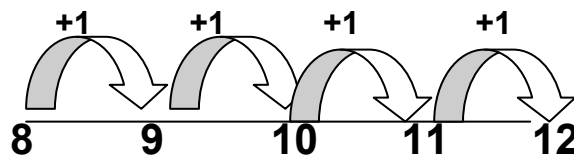
Use in conjunction with a **100 square** to show jumps of tens and ones.

### Counting on to find a small difference

Introduce complementary addition to find differences (only use for **small** differences). The use of models is extremely important here to understand the idea of "difference" (see Y1 guidance).

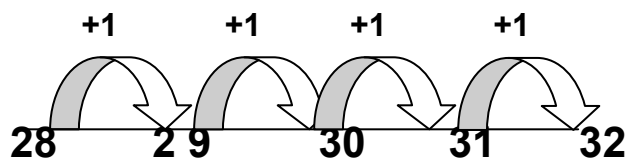
Count up from the smallest number to the largest to **find the difference**.

$$12 - 8 = 4$$



'The difference between 8 and 12 is 4.'

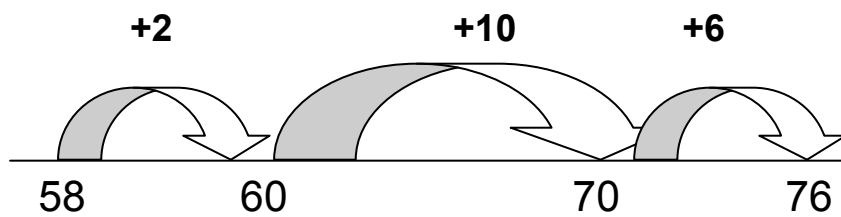
$$32 - 28 = 4$$



'The difference between 28 and 32 is 4.'

If children are confident, further develop this method:

$$76 - 58 = 18$$



'The difference between 58 and 76 is 18.'

Further develop subtraction with numbers that bridge 100, using a **200 grid** to support.

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

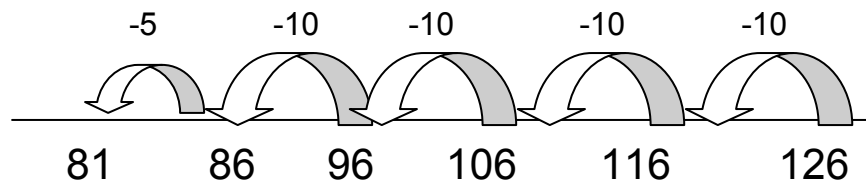
## Subtraction - Year Three

- Subtract numbers with up to three digits, using formal written method of columnar subtraction

**NB** Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

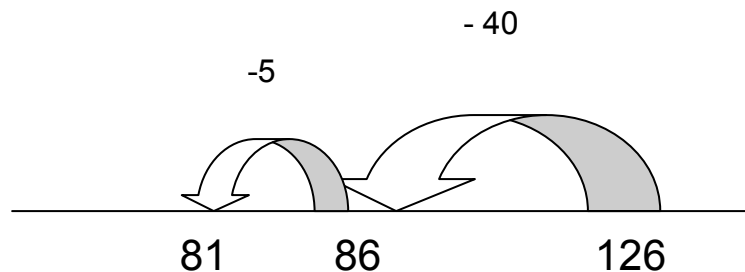
Further develop the use of the **empty number line** with calculations that **bridge 100**:

$$126 - 45 = 81$$



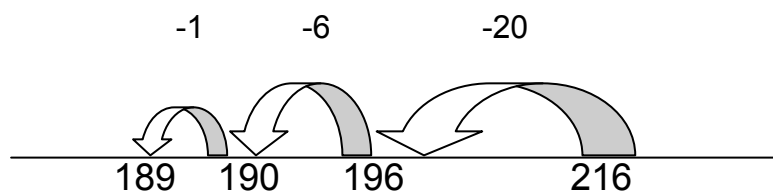
Use a **200 grid** to support counting back in tens and bridging 100

Then use more efficient jumps:



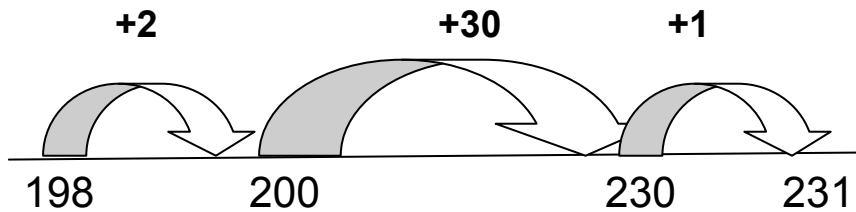
Extend with larger numbers by counting back...

$$216 - 27 = 189$$



...and by **counting on to find the difference** (small difference):

$$231 - 198 = 33$$



'The difference between 198 and 231 is 33.'

Introduce the **expanded written method** with the calculation presented both horizontally and vertically (in columns). Use two-digit numbers when introducing this method, initially:

$$78 - 23 = 55$$

$$\begin{array}{r} 70 + 8 \\ -20 + 3 \\ \hline 50 + 5 = 55 \end{array}$$

'Partition numbers into tens and ones/units.  
Subtract the ones, and then subtract the tens.  
Recombine to give the answer.'

**NB** In this example decomposition (exchange) is not required.

You might replace the **+ sign** with the word '**and**' to avoid confusion.

This will lead into the **formal written method**:

$$\begin{array}{r} 78 \\ -23 \\ \hline 55 \end{array}$$

Use the language of place value to ensure understanding:  
'Eight subtract three, seventy subtract twenty.'

**NB** A number line would be an appropriate method for this calculation but use two-digit numbers to illustrate the formal written method initially.

Introduce the **expanded written method** where **exchange/decomposition** is required:

$$73 - 27 = 46$$

$$\begin{array}{r} 70 + 3 \\ - 20 + 7 \\ \hline \end{array} \quad \text{becomes} \quad \begin{array}{r} 60 + 13 \\ - 20 + 7 \\ \hline 40 + 6 = 46 \end{array} \quad \begin{array}{l} 73 \text{ is partitioned into } 60+13 \text{ in} \\ \text{order to calculate } 73-27 \end{array}$$

**NB** children will need to practise partitioning numbers in this way. **Base- ten materials** could be used to support this.

**When children are confident** with the expanded method introduce the **formal written method**, involving decomposition/exchange:

$$73 - 27 = 46$$

$$\begin{array}{r} 6 \ 13 \\ 7 \ 3 \\ - 2 \ 7 \\ \hline 4 \ 6 \end{array}$$

Use the language of place value to ensure understanding.

'We can't subtract seven from three, so we need to exchange a ten for ten ones to give us 60 + 13.'

Use **base ten materials** to support understanding.

**If children are confident**, extend the use of the **formal written method with numbers over 100**, returning to the expanded method first, if necessary.

$$235 - 127 = 108$$

$$\begin{array}{r} 2 \ 3 \ 5 \\ - 1 \ 2 \ 7 \\ \hline 1 \ 0 \ 8 \end{array}$$

Use the language of place value to ensure understanding.

In this example it has only been necessary to exchange from the tens column.

Use base ten materials to support understanding.

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

## Subtraction - Year Four

- **Subtract numbers with up to 4 digits using the formal written method of columnar subtraction where appropriate**

**NB** Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to teach the use of **empty number lines** with three and four digit numbers, as appropriate.

Continue to develop the **formal written method of subtraction** by revisiting the **expanded method** first, if necessary. Continue to use **base -ten materials** to support understanding.

$$258 - 73 = 185$$

$$\begin{array}{r} 200 + 50 + 8 \\ - \quad 70 + 3 \\ \hline \end{array} \quad \text{becomes} \quad \begin{array}{r} 100 + 150 + 8 \\ - \quad 70 + 3 \\ \hline 100 + 80 + 5 = 185 \end{array}$$

You might replace the **+** sign with the word '**and**' to avoid confusion. Children will need to practise partitioning in a variety of ways.

This leads to the **formal written method**, involving decomposition...

$$\begin{array}{r} 1 \ 15 \\ 2 \ 5 \ 8 \\ - \quad 7 \ 3 \\ \hline 1 \ 7 \ 5 \end{array}$$

Use the language of place value to ensure understanding. In this example it has been necessary to exchange from the hundreds column.

Further develop by subtracting a three-digit number from a three-digit number:

$$637 - 252 = 385$$

$$\begin{array}{r} 600 + 30 + 7 \\ - \quad 200 + 50 + 2 \\ \hline \end{array} \quad \begin{array}{r} 500 + 130 + 7 \\ - \quad 200 + 50 + 2 \\ \hline 300 + 80 + 5 = 385 \end{array}$$

Ensure that children are confident in partitioning numbers in this way.

This leads to a **formal written method**:

$$\begin{array}{r} \phantom{5} \phantom{13} \\ \mathbf{637} \\ - \mathbf{252} \\ \hline \mathbf{385} \end{array}$$

Use the language of place value to ensure understanding and use base-ten materials, if necessary.

**When children are confident**, develop with **four digit numbers** and decimal numbers (in the context of money and measures).

$$\mathbf{3625 - 1219 = 2406}$$

$$\begin{array}{r} \phantom{1} \phantom{15} \\ \mathbf{3625} \\ - \mathbf{1219} \\ \hline \mathbf{2406} \end{array}$$

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

## Year Five - Subtraction

- **Subtract whole numbers with more than 4 digits, including using formal written method (columnar subtraction)**

**NB** Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to teach the use of **empty number lines** with larger numbers and decimals, as appropriate.

Continue to develop the **formal written method for subtraction** with three and four digit numbers (see Y4 guidance), returning to an expanded method and using base ten materials, if necessary.

$$503 - 278 = 225$$

$$\begin{array}{r} 500 + 0 + 3 \\ - 200 + 70 + 8 \\ \hline \end{array} \quad \begin{array}{r} 400 + 90 + 13 \\ - 200 + 70 + 8 \\ \hline 200 + 20 + 5 \end{array}$$

In this example 503 has to be partitioned into 400+90+13 in order to carry out the subtraction calculation.

This leads into the **formal written method** (there is potential for error in this example):

$$\begin{array}{r} \phantom{4} \phantom{9} \phantom{13} \\ \phantom{4} \phantom{9} \phantom{13} \\ \phantom{4} \phantom{9} \phantom{13} \\ - 278 \\ \hline 225 \end{array}$$

There are no tens in the first number (503) so we have to exchange a hundred for 10 tens before we can exchange a ten for ten ones/units

**NB** It would be appropriate to discuss the use of mental calculation methods with an example like this one, i.e. would an empty number line be a more efficient method for these numbers?

**When children are confident** extend with larger numbers (and decimal numbers). Return to an expanded method, if necessary.

$$12731 - 1367 = 11364$$

$$\begin{array}{r} \phantom{1}^6 \phantom{2}^{12} \phantom{7}^{11} \\ 12731 \\ - \quad 1367 \\ \hline 11364 \end{array}$$

In this example it has been necessary to exchange from the tens and the hundreds columns.

**NB** If children are making significant errors, provide calculations where only one exchange is required.

Introduce subtraction of decimals, initially in the context of money and measures.

$$£166.25 - £83.72 = £82.53$$

$$\begin{array}{r} \phantom{1}^{16} \phantom{6}^5 \phantom{6}^{12} \\ 166.25 \\ - \quad 83.72 \\ \hline 82.53 \end{array}$$

Ensure the decimal points line up.

Continue to practise and apply the formal written method with large numbers and decimals throughout year five.

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

### Year Six - Subtraction

No objectives have been included in the programmes of study explicitly related to written methods for subtraction in Y6. However, there is an expectation that children will continue to practice and use **the formal written method for larger numbers and decimals** and use these methods when solving problems, when appropriate (see previous years' guidance for methods).

Our aim is that by the end of Y6 children **use mental methods (with jottings)** when appropriate, but for calculations that they cannot do in their heads, they use an efficient **formal written method** accurately and with confidence.

## Stages in Multiplication

### Multiplication – Early Stages (EYFS)

Children will engage in a wide variety of songs and rhymes, games and activities. In practical activities and through discussion they will begin to solve problems involving doubling.



‘Three apples for you and three apples for me. How many apples altogether?’

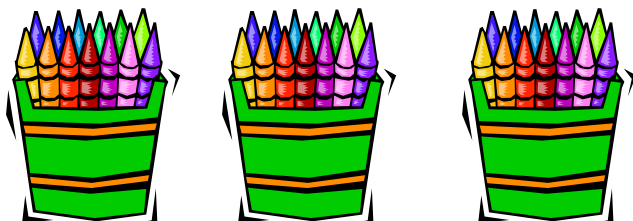
### Multiplication – Year One

- **Solve one-step problems involving multiplication by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher**
- **Count in multiples of twos, fives and tens (to the 10<sup>th</sup> multiple)**

Children will count repeated groups of the same size in practical contexts. They will use the vocabulary associated with multiplication in practical contexts. They will solve **practical problems** that involve combining groups of 2, 5 or 10. e.g. socks, fingers and cubes.

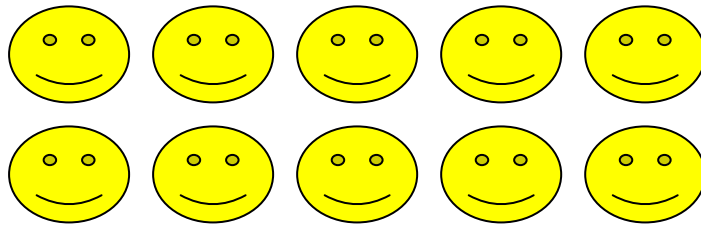


‘Six pairs of socks.  
How many socks altogether? 2, 4, 6, 8, 10, 12’

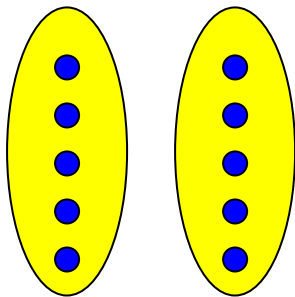


‘Three pots of ten crayons. How many crayons altogether? 10, 20, 30’

Use **arrays** to support early multiplication



'Five groups of two faces. How many faces altogether? 2, 4, 6, 8, 10'  
Two groups of five faces. How many faces altogether? 5, 10'



'2 groups of 5'

'How many altogether?'

'5 + 5 = 10'

Double five is ten

Continue to solve problems **in practical contexts** and develop the language of early multiplication, with appropriate resources, throughout Y1.

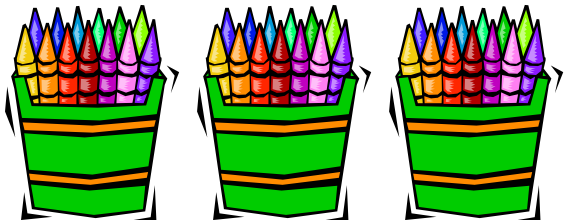
## Multiplication - Year Two

- Recall and use multiplication facts for the 2, 5 and 10 multiplication tables
- Calculate mathematical statements for multiplication within the multiplication tables and write them using the multiplication ( $\times$ ) and equals (=) signs
- solve problems involving multiplication, using materials, arrays, repeated addition, mental methods, and multiplication facts, including problems in contexts
- show that multiplication of two numbers can be done in any order (commutative)

**NB** Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Children will use a range of vocabulary to describe multiplication and use practical resources, pictures, diagrams and the x sign to record.

### Combining Groups (repeated addition):



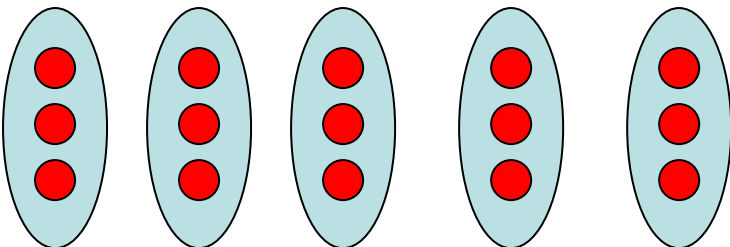
'3 groups of 10 crayons'

'How many crayons altogether?'

' $10 + 10 + 10 = 30$ '

'3 groups of 10' '3 times ten'

' $3 \times 10 = 30$ ' ' $10 \times 3 = 30$ '



'5 groups of 3' '5 lots of 3' ' $3 + 3 + 3 + 3 + 3 = 15$ '

'5 times 3' '3 multiplied by 5' ' $5 \times 3 = 15$ ' ' $3 \times 5 = 15$ '

Using arrays to support multiplication:

$$6 \times 5 = 30$$



$$'5 + 5 + 5 + 5 + 5 + 5 = 30'$$

'6 rows of 5'

'6 groups of 5'

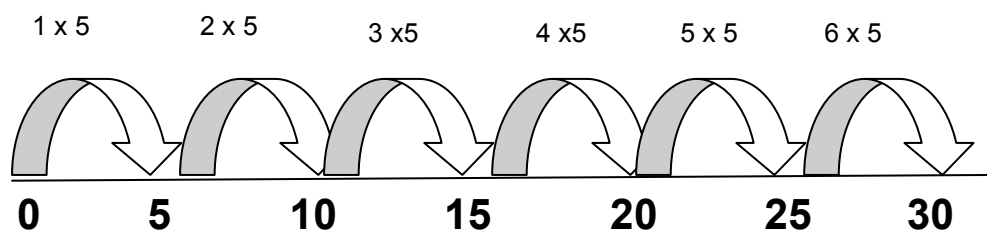
'5 groups of 6'

$$'5 \times 6 = 30'$$

$$'6 \times 5 = 30'$$

Using an empty number line:

$$6 \times 5 = 30$$



Make the link to repeated addition.

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

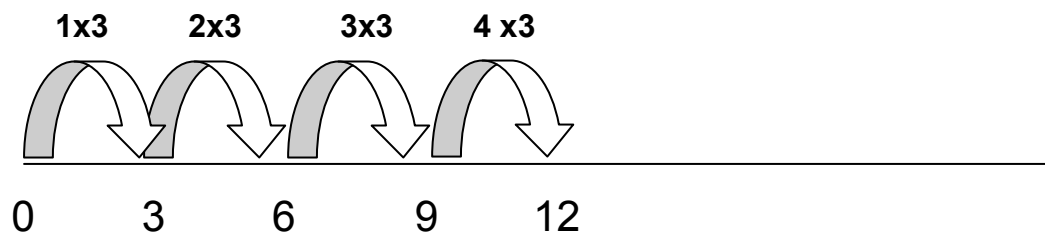
## Multiplication - Year Three

- **Recall and use multiplication facts for the 3, 4 and 8 multiplication tables** (continue to practise the 2, 5 and 10 multiplication tables)
- **Write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to a formal written method**

**NB** Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to use **number lines** and **arrays** to support multiplication, as appropriate (see Y2 guidance).

$$4 \times 3 = 12$$



**Partitioning method for multiplication of a teen number by a one-digit number:**

$$13 \times 5 = 65 \quad (\text{Partition } 13 \text{ into } 10 + 3)$$

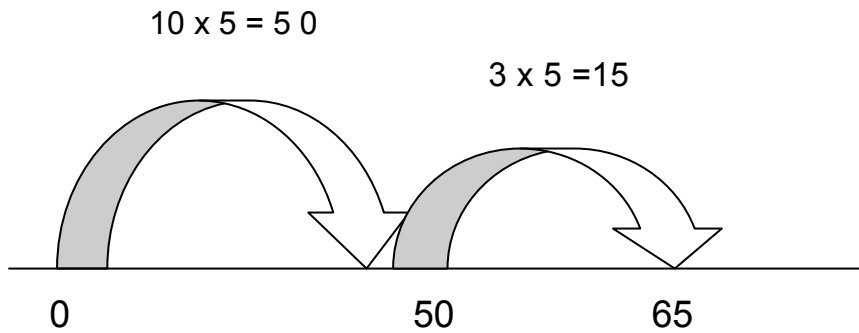
$$10 \times 5 = 50$$

$$3 \times 5 = 15$$

$$50 + 15 = 65$$

Demonstrate the partitioning method using a **number line**:

$$13 \times 5 = 65$$



**Grid Method** (teen number multiplied by a one- digit number):

$$13 \times 8 = 104$$

X	10	3
8	80	24

$$80 + 24 = 104$$

'Partition 13 into  $10 + 3$  then multiply each number by 8. Add the partial products (80 and 24) together.'

This will lead into **expanded short multiplication**:

$$13 \times 8 = 104$$

$$\begin{array}{r} 10 + 3 \\ \times \quad 8 \\ \hline 24 \quad (3 \times 8) \\ + 80 \quad (10 \times 8) \\ \hline 104 \end{array}$$

Include an addition symbol when adding partial products.

Refine the recording in **preparation for formal short multiplication**:

$$13 \times 8 = 104$$

$$\begin{array}{r} 13 \\ \times 8 \\ \hline 24 \quad (3 \times 8) \\ + 80 \quad (10 \times 8) \\ \hline 104 \end{array}$$

Use the language of place value to ensure understanding.

Include an addition symbol when adding partial products.

Model the same calculation using a number line, if necessary, to ensure understanding.

**Formal short multiplication:**

$$\begin{array}{r} 13 \\ \times 8 \\ \hline 104 \\ \phantom{104} \phantom{2} \end{array}$$

Ensure that the digit 'carried over' is written under the line in the correct column.

Use the language of place value to ensure understanding.

Continue to develop the formal written method of multiplication throughout year three using teen- numbers multiplied by a one-digit number.

**If children are confident** progress to multiplying other two-digit numbers by a one-digit number (see Y4 guidance).

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

## Multiplication- Year Four

- Recall multiplication facts for multiplication tables up to  $12 \times 12$
- Multiply two-digit and three-digit numbers by a one-digit number using formal written layout

**NB** Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to use **empty number lines**, as appropriate (see Y3 guidance).

Further develop the **grid method** for two-digit numbers multiplied by a one-digit number.

$$36 \times 4 = 144$$

X	30	6
4	120	24

$$120 + 24 = 144 \text{ (add the partial products)}$$

**Expanded short multiplication** (two-digit number by a one-digit number):

$$36 \times 4 = 144$$

$$\begin{array}{r} 30 + 6 \\ \times \quad 4 \\ \hline 24 \\ + 120 \\ \hline 144 \end{array}$$

$$(4 \times 6 = 24)$$
$$(4 \times 30 = 120)$$

Include an addition symbol when adding partial products.

Refine the recording in preparation for formal short multiplication:

$$36 \times 4 = 144$$

$$\begin{array}{r} 36 \\ \times 4 \\ \hline + 24 \quad (4 \times 6) \\ \hline 120 \quad (4 \times 30) \\ \hline 144 \end{array}$$

This leads to **short multiplication (formal method)** of a two-digit number multiplied by a one-digit number:

$$36 \times 4 = 144$$

$$\begin{array}{r} 36 \\ \times 4 \\ \hline 144 \\ \hline \end{array}$$

Use the language of place value to ensure understanding.  
Ensure that the digit 'carried over' is written under the line in the correct column.

Continue to practise the formal method of short multiplication of a two-digit number by a one-digit number throughout Y4.

**If children are confident**, continue to develop short multiplication with three-digit numbers multiplied by a one-digit number.

If necessary, return to **the grid method and/or expanded method** first:

$$127 \times 6 = 762$$

<b>x</b>	100	20	7
6	600	120	42

$$600 + 120 + 42 = 762 \quad (\text{add the partial products})$$

This leads to **expanded short multiplication**:

$$127 \times 6 = 762$$

$$\begin{array}{r} 127 \\ \times 6 \\ \hline 42 \quad (6 \times 7) \\ + 120 \quad (6 \times 20) \\ \hline 600 \quad (6 \times 100) \\ \hline 762 \end{array}$$

This will lead into **short multiplication (formal method)**:

$$\begin{array}{r} 127 \\ \times 6 \\ \hline 762 \\ \hline \end{array}$$

Use the language of place value to ensure understanding.

Ensure that the digits 'carried over' are written under the line in the correct column.

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

## Multiplication - Year Five

- **Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers**

**NB** Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Build on the work covered in Y4 with the **formal method of short multiplication** (two-digit number multiplied by a one-digit number).

**When children are confident introduce multiplication by a two-digit number.** If necessary, return to the grid method and/or expanded method first.

**Grid method** (two-digit number multiplied by a teen- number):

$$23 \times 13 = (20 + 3) \times (10 + 3) = 299$$

X	20	3
10	200	30
3	60	9

$$\begin{array}{r} 230 \\ + 69 \\ \hline 299 \end{array}$$

Add the partial products  $(200 + 30) + (60 + 9) = 299$

**Expanded long multiplication** (two-digit numbers multiplied by a teen- number):

$$23 \times 13 = 299$$

$$\begin{array}{r} 23 \\ \times 13 \\ \hline 9 \quad (3 \times 3) \\ 60 \quad (3 \times 20) \\ + 30 \quad (10 \times 3) \\ \hline 200 \quad (10 \times 20) \\ \hline 299 \end{array}$$

This leads into...

**Compact long multiplication (formal method):**

$$23 \times 13 = 299$$

$$\begin{array}{r} 23 \\ \times 13 \\ \hline + 69 \quad (3 \times 23) \\ 230 \quad (10 \times 23) \\ \hline \underline{299} \end{array}$$

Use the language of place value to ensure understanding.

Add the partial products.

Extend to larger two-digit numbers:

$$56 \times 27 = (50 + 6) \times (20 + 7) = 1512$$

<b>x</b>	50	6	
20	1000	120	1120
7	350	42	392
			1512

Add the partial products  $(1000 + 120) + (350 + 42) = 1512$

**Expanded long multiplication (two-digit numbers multiplied by two-digit numbers):**

$$56 \times 27 = 1512$$

$$\begin{array}{r} 56 \\ \times 27 \\ \hline 42 \quad (7 \times 6) \\ 350 \quad (7 \times 50) \\ + 120 \quad (20 \times 6) \\ \hline 1000 \quad (20 \times 50) \\ \hline 1512 \\ \hline \end{array}$$

This expanded method is linked to the grid method

This leads into...

**Compact long multiplication (formal method):**

**56 x 27 = 1512**

$$\begin{array}{r} 56 \\ \times 27 \\ \hline 392 \\ + 1120 \\ \hline 1512 \\ \hline 1 \end{array}$$

Use the language of place value to ensure understanding.

In this example there are digits that have been 'carried' over in the partial products.

Add the partial products.

**When children are confident** with long multiplication extend with three-digit numbers multiplied by a two-digit number, returning to the grid method first, if necessary:

**124 x 26 = 3224**

$$\begin{array}{r} 124 \\ \times 26 \\ \hline 744 \\ + 2480 \\ \hline 3224 \\ \hline 11 \end{array}$$

Use the language of place value to ensure understanding.

Add the partial products.

The prompts (in brackets) can be omitted if children no longer need them.

Extend with short and long multiplication of decimal numbers (initially in the context of money and measures), returning to an expanded method first, if necessary (see Y6 guidance).

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

## Multiplication - Year Six

- **Multiply multi-digit numbers (including decimals) up to 4 digits by a two-digit whole numbers**

**NB** Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to practise and develop the **formal short multiplication** method and **formal long multiplication** method with larger numbers and decimals throughout Y6. Return to an expanded forms of calculation initially, if necessary (see Y5 guidance).

**The grid method** (decimal number multiplied by a two-digit number):

$$53.2 \times 24 = 1276.8$$

x	50	3	0.2	
20	1000	60	4	1064.0
4	200	12	0.8	212.8
				1276.8

**The formal written method of long multiplication:**

$$\begin{array}{r} 53.2 \\ \times 24.0 \\ \hline 212.8 \\ 1064.0 \\ \hline 1276.8 \end{array}$$

(53.2 x 4)  
(53.2 x 20)

It is an option to include .0 in this example, but not essential.

The prompts (in brackets) can be omitted if children no longer need them.

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

Our aim is that by the end of Y6 children use **mental methods (with jottings)** when appropriate, but for calculations that they cannot do in their heads, they use an efficient **formal written method** accurately and with confidence.

## Stages in Division

### Division – Early stages (EYFS)

Children will engage in a wide variety of songs and rhymes, games and activities. In practical activities and through discussion they will begin to solve problems involving halving and sharing.



Share the apples between two people.

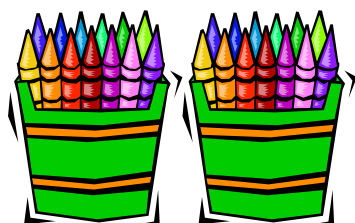
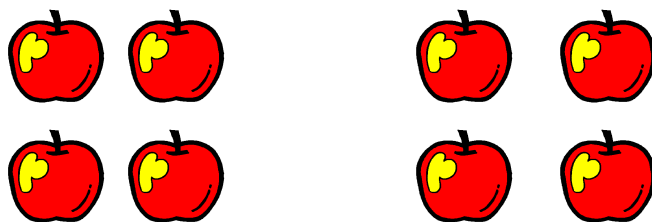
'Half of the apples for you and half of the apples for me.'

### Division - Year One

- **Solve one-step problems involving division by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher**
- **Count in multiples of twos, fives and tens (to the 10<sup>th</sup> multiple)**

Children will start with practical **sharing** using a variety of resources. They will share objects into **equal groups** in a variety of situations. They will begin to use the vocabulary associated with division in practical contexts.

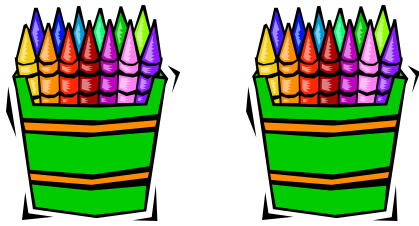
'Share these eight apples equally between two children. How many apples will each child have?'



'Share 20 crayons between 2 pots.'

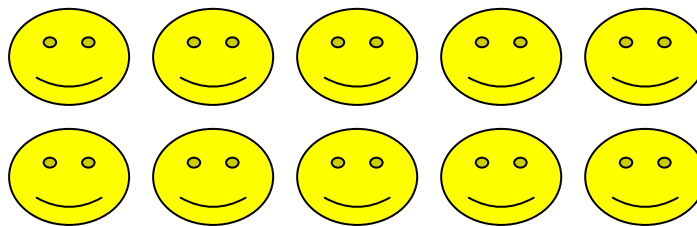
'How many crayons are in each pot?'

Children will move from **sharing** to **grouping** in a practical way

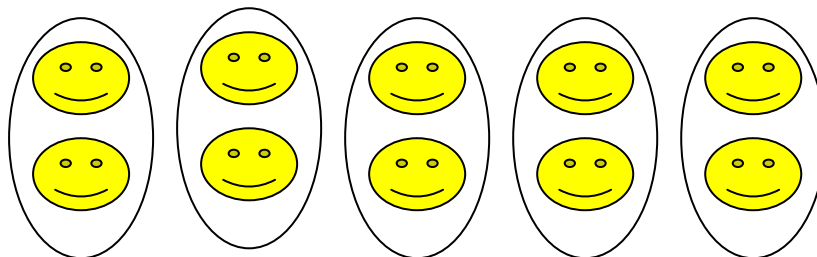


'Put 20 crayons into groups of 10. How many pots do we need?'

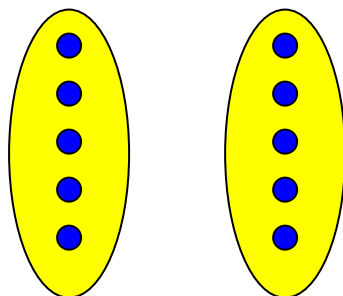
Use **arrays** to support early division



'How many faces altogether? How many groups of two?'



'Five groups of two'



'How many groups of 5?'  
'10 shared equally between 2 people'  
'Half of ten is five'

Continue to solve problems in **practical contexts** throughout Y1, and develop the language of early division, with appropriate resources.

## Division - Year Two

- Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables
- Calculate mathematical statements for division within the multiplication tables they know and write them using the division ( $\div$ ) and equals (=) signs
- Solve problems involving division, using materials, arrays, repeated subtraction, mental methods, and multiplication and division facts, including problems in contexts

**NB** Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Children will use a range of vocabulary to describe division and use practical resources, pictures, diagrams and the  $\div$ -sign to record, using multiples that they know.

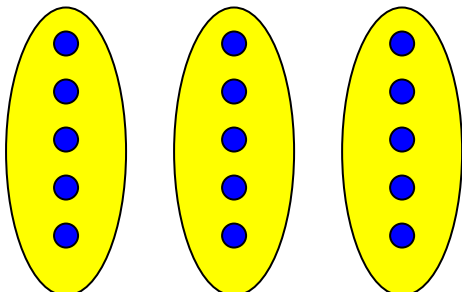
### Sharing and grouping:



'30 crayons shared equally between three pots.' (Sharing)  
'We have 30 crayons and put ten crayons in each pot.  
How many pots do we need?' (Grouping)

'30 divided by 10 = 3'  
'30 divided by 3 = 10'

$$30 \div 10 = 3$$
$$30 \div 3 = 10$$



'How many groups of 5?'  
'15 shared equally between 3 people  
is...?'

'15 divided by 3 equals 5'  
'15 divided by 5 equals 3'

$$15 \div 5 = 3$$
$$15 \div 3 = 5$$

## Using arrays to support division

$$15 \div 5 = 3$$

$$15 \div 3 = 5$$



How many groups of 3?

How many groups of 5?

15 shared between 3 people is...?

15 shared between 5 people is...?

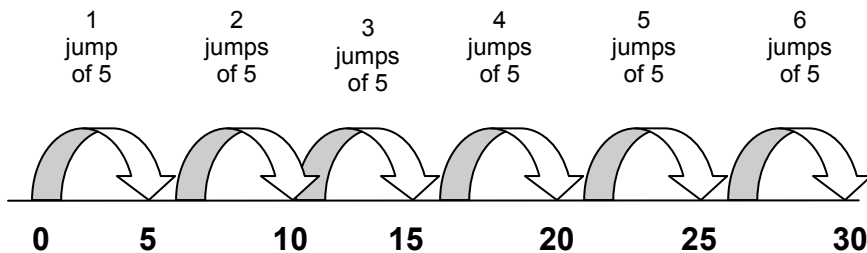
15 divided by 5 = 3

15 divided by 3 = 5

When children are ready, use an empty number line to count forwards:

$$30 \div 5 = 6$$

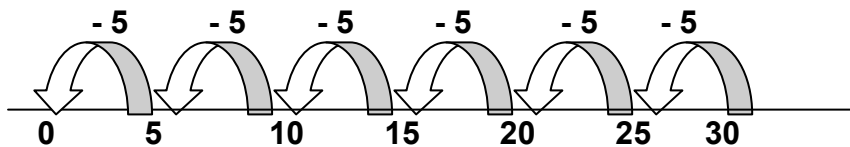
'How many jumps of five make thirty?'



Also jump back to make the link with repeated subtraction:

$$30 \div 5 = 6$$

'How many groups of five?'



**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

## Year Three – Division

- **Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables** (continue to practise the 2, 5 and 10 multiplication tables)
- **Write and calculate mathematical statements for division using the multiplication tables that they know, including for two-digit numbers divided by one-digit numbers, using mental and progressing to a formal written method**

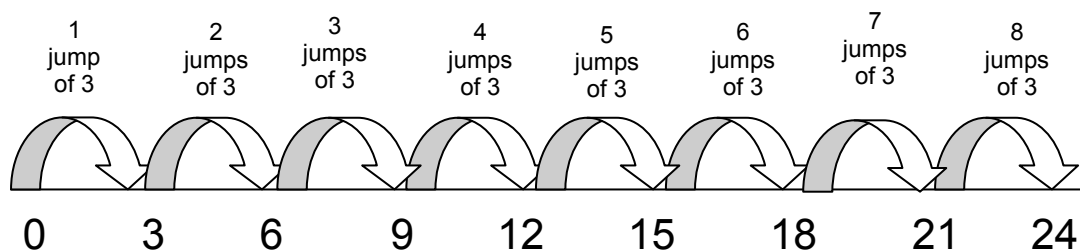
**NB** Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to use practical resources, pictures, diagrams, number lines, arrays and the  $\div$  sign to record, using multiples that they know, as appropriate (see Y2 guidance).

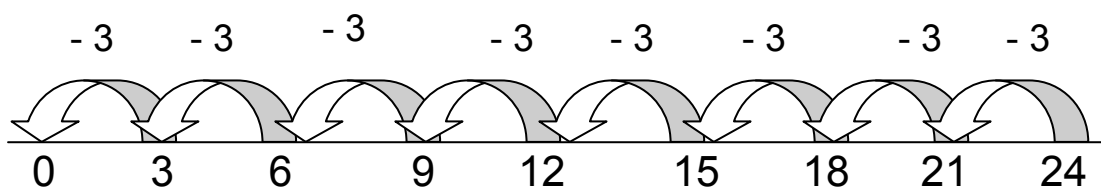
**Using an empty number line to count forwards...**

$$24 \div 3 = 8$$

'How many threes in 24?'



**...also jump back from 24 to make the link with repeated subtraction.**



'How many groups of three in 24?'

**Introduce the formal layout using multiplication/division facts that the children know:**

$$24 \div 3 = 8$$

This can also be recorded as...

$$\begin{array}{r} 8 \\ \hline 3 \overline{) 24} \end{array}$$

'Twenty four divided by three equals eight.'

'How many threes are there in twenty four?'

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

## Year Four- Division

- Recall multiplication and division facts for multiplication tables up to  $12 \times 12$
- Use place value, known and derived facts to divide mentally
- Divide two-digit and three-digit numbers by a one-digit number using formal written layout (not explicitly stated in the programmes of study but implied in the non-statutory guidance)

**NB** Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

Continue to write and calculate mathematical statements for division using the multiplication tables that the children know e.g.

$$32 \div 8 = 4$$

Continue using the **formal written layout** for division using multiplication tables that they know:

$$\begin{array}{r} 4 \\ \hline 8 \overline{) 32} \end{array}$$

'How many eights are there in thirty two?'

Continue using the formal written layout, introducing remainders:

$$25 \div 3 = 8 \text{ r } 1$$

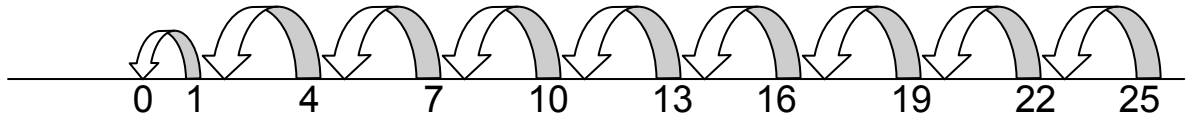
$$\begin{array}{r} 8 \text{ r } 1 \\ \hline 3 \overline{) 25} \end{array}$$

**NB** Remainders are not specifically referred to until Y5 in the National Curriculum. However, this may be an appropriate point to introduce them using familiar multiplication facts.

This could be modelled using an empty number line, if necessary:

'Eight jumps of three and one left over.'

$$25 \div 3 = 8 \text{ r}1$$



Alternatively you could jump forwards in multiples of three from zero to twenty four ('and one more makes 25')

**Division using partitioning** (two digits divided by one digit):

$$65 \div 5 = 13$$

$$65 = 50 + 15 \quad \text{Partition 65 into 50 and 15}$$

$$50 \div 5 = 10$$

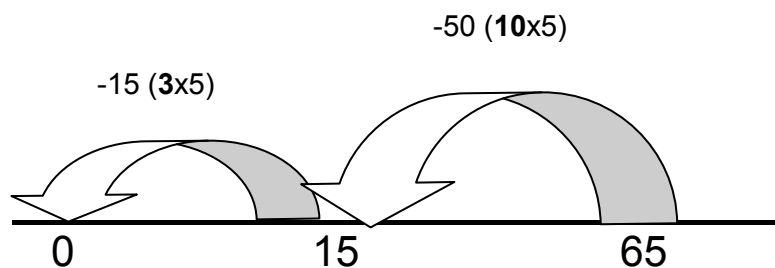
$$15 \div 5 = 3$$

$$10 + 3 = 13$$

**NB** Children will need to practise partitioning in a variety of ways.

Continue to use **empty number lines**, as appropriate, using multiples of the divisor:

$$65 \div 5 = 13$$



$$98 \div 7 = 14$$

$$98 = 70 + 28 \quad \text{Partition 98 into 70 and 28}$$

$$70 \div 7 = 10$$

$$28 \div 7 = 4$$

$$10 + 4 = 14$$

This could be modelled on an empty number line to further develop understanding.

**NB** Children will need to practise partitioning in a variety of ways.

$$98 \div 7 = 14$$

'We have partitioned 98 into 70 and 28 (90 = 70 + 28).

Seven 'goes into' 70 ten times and seven 'goes into' 28 four times.  
Ten add four equals 14'

$$\begin{array}{r} 10 + 4 = 14 \\ 7 \overline{) 70 + 28} \end{array}$$

This will lead into the formal written method of short division:

$$98 \div 7 = 14$$

$$\begin{array}{r} 14 \\ 7 \overline{) 98} \end{array}$$

Use the vocabulary of place value to ensure understanding and make the link to partitioning.

Continue to practise the formal method of short division throughout Y4.

**If children are confident** develop further, by dividing three-digit numbers by a one-digit number using the formal method of short division with whole number answers (no remainders).

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

## Year Five - Division

- Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context

**NB** Ensure that children are confident with the methods outlined in the previous year's guidance before moving on.

**Continue to practise the formal written method of short division with whole number answers...**

$$184 \div 8 = 23$$

$$\begin{array}{r} 23 \\ 8 \overline{) 184} \end{array}$$

Use the language of place value to ensure understanding.

Make the link to the partitioning method (see Y4 guidance).

...and with remainders:

$$432 \div 5 = 86 \text{ r}2$$

$$\begin{array}{r} 86 \text{ r}2 \\ 5 \overline{) 432} \end{array}$$

The remainder can also be expressed as a fraction,  $\frac{2}{5}$  (the remainder divided by the divisor):  $432 \div 5 = 86\frac{2}{5}$

Continue to practise, develop and extend the formal method of short division, with and without remainders. Interpret and express remainders according to the context.

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

## Year Six – Division

- Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context
- Divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context

**NB** Ensure that children are confident with the methods outlined in the previous year's guidance before moving on

Continue to practise the **formal method of short division**, with and without remainders, using the language of place value to ensure understanding (see Y5 guidance).

$$496 \div 11 = 45 \text{ r}1$$

$$\begin{array}{r} 45 \text{ r}1 \\ 11 \overline{) 496} \end{array}$$

The remainder can also be expressed as a fraction,  $\frac{1}{11}$  (the remainder divided by the divisor)

Dividing by a two-digit number using a **formal method of long division**:

$$\begin{array}{r} 45 \text{ r}1 \\ 11 \overline{) 496} \\ \underline{- 440} \quad (40 \times 11) \\ 56 \\ \underline{- 55} \quad (5 \times 11) \\ 1 \text{ (remainder)} \end{array}$$

Multiples of the divisor (11) have been subtracted from the dividend (496)

'40 (lots of 11) + 5 (lots of 11) = 45 (lots of 11)'

'1 is the remainder'

Answer:  $45\frac{1}{11}$

Standard short division does not help with the following calculation. However, it can be solved using **long division** (by repeated subtraction using multiples of the divisor):

$$144 \div 16 = 9$$

$$\begin{array}{r}
 \phantom{0}9 \\
 16 \overline{) 144} \\
 \underline{- 64} \quad (4 \times 16) \\
 80 \\
 \underline{- 64} \quad (4 \times 16) \\
 16 \\
 \underline{- 16} \quad (1 \times 16) \\
 0
 \end{array}$$

Multiples of the divisor (16) have been subtracted from the dividend (144)

'4 (lots of 16) + 4 (lots of 16) + 1 (lot of 16) = 9 (lots of 16)

There is no remainder'

Children will need to select the most effective method for each calculation/problem they meet, including whether to use the standard, **formal written method of long division**:

$$432 \div 15 = 28 \text{ r}12$$

$$\begin{array}{r}
 \phantom{0}28 \text{ r}12 \\
 15 \overline{) 432} \\
 \underline{300} \quad (20 \times 15) \\
 132 \\
 \underline{120} \quad (8 \times 15) \\
 12 \quad (\text{remainder})
 \end{array}$$

Multiples of the divisor (15) have been subtracted from the dividend (432)

'20 (lots of 15) + 8 (lots of 15) = 28

12 is the remainder'

The remainder can also be expressed as a fraction,  $\frac{12}{15}$  (the remainder divided by the divisor) or as a decimal, **0.8** (see next example)

The answer is:  $28 \frac{12}{15}$  or 28.8

This is an alternative way of recording formal long division:

$$432 \div 15 = 28.8$$

$$\begin{array}{r} 28.8 \\ \hline 15 \overline{) 432.0} \\ \underline{30} \phantom{0} \\ 132 \\ \underline{120} \\ 120 \\ \underline{120} \\ 0 \end{array}$$

**NB** Only teach this method when children are completely secure with the previous method.

The remainder is expressed as a decimal.

**NB** If, at any time, children are making significant errors, return to the previous stage in calculation.

Our aim is that by the end of Y6 children use **mental methods (with jottings)** when appropriate, but for calculations that they cannot do in their heads, they use an efficient **formal written method** accurately and with confidence.

**NOTES**