

# St Cuthbert's Catholic Primary School



## Calculation Policy

Date Adopted:	
Person Responsible:	Mrs E Hardman & Mrs S McAuley Subject Leaders

**Signed:** ..... (Person Responsible) **Date:** .....

**Signed:** ..... (Headteacher) **Date:** .....

**Signed:** ..... (Chair/Governor) **Date:** .....

## School Mission statement

With Jesus we love, learn and pray; to grow in the St Cuthbert's way.

## Our Values

Loving Honest

Charitable Respectful

Aspirational Faithful

Positive Forgiving

## School's Aims

**To proclaim Jesus Christ as the centre of our school community.**

*"Trust in the Lord with all your heart and lean not on your own understanding; in all your ways submit to him, and he will make your paths straight." Proverbs 3:5*

**To enjoy times of prayer and worship together.**

*"Our Father in heaven, hallowed be your name, thy kingdom come, your will be done, on earth as it is in heaven." Matthew 9:10*

**To be a vibrant place; where we can all feel loved, and all visitors are welcomed with warmth and respect. Recognising that everybody is created in the 'image and likeness of God.'**

*"Welcome one another as Christ has welcomed you, for the glory of God." Romans 15:7*

**To promote; a love of learning, an enthusiasm to succeed and aspirations to 'dream big'.**

*"Commit your work to the Lord, and your plans will be established." Proverbs 16:3*

**To ensure each and every one of us has the opportunity to thrive; to share the 'gifts' we have been blessed with, and to achieve to our full potential in all that we do.**

**"I can do all this through him who gives me strength." Phillipians 4:13**

**To feel safe, respected, loved and valued, so we have the confidence and resilience to 'make a difference.'**

*"God is our refuge and strength, an ever-present help in trouble." Psalm 46.1ning light in our parish,*

**To be a shining light in our parish, in our community and in our world.**

*For where two or three are gathered in my name, there I am among them." Matthew 18:20*

**Be a place where it is ok to make mistakes. Forgiveness makes every day a new beginning.**

*"Forgive and you will be forgiven." Luke 6:37*

**St Cuthbert's has adapted the Lancashire Mathematics Team's revised 'Progression Towards Written Methods of Calculation Policy' during work with the WOWS Improvement Cluster supported by Sarah Martin (Spring/Summer Term 2014).**

Please note:

Although many of the KS2 objectives suggest that children should be using formal written methods, the National Curriculum document states:

*"The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study."* p4

It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

There is no statutory requirement for particular methods of calculation to be used in schools. The appendix of calculations sets out some examples of formal written methods for all four operations to illustrate the range of methods that **could** be taught. It is **not intended** to be an exhaustive list, nor is it intended to show progression in formal written methods. p128

Children will progress through the stages of mental to written methods for each calculation at a pace appropriate to their development and understanding.

At all stages it is vital that children are provided with concrete, pictorial and symbolic representations of calculations and this is emphasised throughout the policy. It is important that children do not abandon jottings and mental methods once written methods are introduced. Therefore children will always be encouraged to look at a calculation and then decide which is the best method to use – pictorial, mental calculation with or without jottings or structured recording.

Our aim is to develop children who can:

- Choose the most efficient method for the task;
- Calculate accurately with understanding of the value of numbers;
- Show confidence in their calculation work;
- Recognise the relevance of calculations in everyday life;
- Apply their calculation skills in a range of contexts.

Susan McAuley  
October 2014

## PROGRESSION TOWARDS A WRITTEN METHOD FOR ADDITION

In developing a written method for addition, it is important that children understand the concept of addition, in that it is:

- Combining two or more groups to give a total or sum
- Increasing an amount
- Adding whole, decimal and negative numbers

Also, to understand and work with certain principles:

- the inverse of subtraction
- commutative i.e.  $5 + 3 = 3 + 5$
- associative i.e.  $5 + 3 + 7 = 5 + (3 + 7)$
- = sign means same value but 'looks different'

**Always ensure that children are provided with an opportunity to access**

- ✓ Concrete
- ✓ Pictorial representation
- ✓ Symbolic

**DO – TALK – RECORD**

The fact that it is commutative and associative means that calculations can be rearranged, e.g.  $4 + 13 = 17$  is the same as  $13 + 4 = 17$ .

### Early Learning Goal

**Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number.**

**Using quantities and objects, they add and subtract two single-digit numbers and count on and back to find the answer.**

**They solve problems including doubling, halving and sharing.**

*Children should experience practical calculation opportunities using a wide variety of **practical equipment**, including small world play, role play, counters, cubes etc. They should be encouraged to develop ways of recording calculations using **pictures** and enhance the learning experience of developing a **mental picture** of the number system in their heads to use for calculation.*

*Initially addition calculations will not cross the tens boundary but as children progress it will cross the tens boundary into teen numbers (as long as there is understanding of quantity value and the five strands of place value).*

There are two main models of addition at this early stage – **aggregation** and **augmentation**.

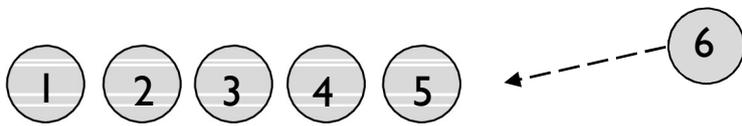
### **Aggregation - combining two sets of objects and counting all method**

Strategy – children count one set, count the other and then count all (stringer).

Children will begin to develop their ability to add by using **practical equipment** to count out the correct amount for each number in the calculation and then combine them to find the total. For example, when calculating  $4 + 2$ , they are encouraged to count out four counters and count out two counters.



To find how many altogether, touch and drag them into a line one at a time whilst counting.



By touch counting and dragging in this way, it allows children to keep track of what they have already counted to ensure they do not count the same item twice.

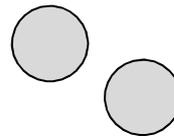
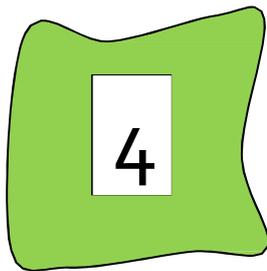
**Augmentation – adding on to a set (counting on method)**

Strategy – requires fluency with counting from any number (chainer).

To support children in moving from a counting all strategy to one involving counting on, children should still have two groups of objects but one should be covered so that it cannot be counted. For example, when calculating  $4 + 2$ , count out the two groups of counters as before.

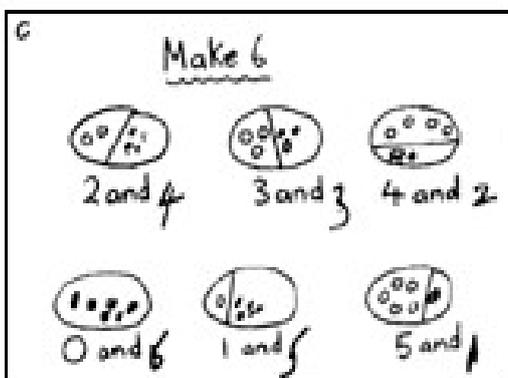


Then cover up the larger group with a cloth.



For most children, it is beneficial to place the digit card on top of the cloth to remind the children of the number of counters underneath. They can then start their count at 4, and touch count 5 and 6 in the same way as before, rather than having to count all of the counters separately as before. Those who are ready may record their own calculations.

- ✓ Ensure that children can partition single-digit numbers in different ways and have opportunity to explore this using different **concrete equipment**.



**Concrete** – counters  
**Pictorial representation** – see drawings  
**Symbolic** – written number sentence  
 Children **who are ready** may record this as:  
 $6 = 2 + 4$     $6 = 3 + 3$     $6 = 4 + 2$   
 $6 = 0 + 6$     $6 = 1 + 5$     $6 = 5 + 1$

- ✓ If children are ready this can be developed to - counting on with a bead bar/number line (see Y1).

### End of Y1 Objectives

Read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs.

Represent and use number bonds and related subtraction facts within 20.

Add and subtract one-digit and two-digit numbers to 20, including zero using concrete and pictorial representations).

Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as  $7 = \square - 9$ .

*Children will continue to use practical equipment e.g. counters to combine groups of **objects** to find the total (aggregation). To become more efficient they will move on to the use of **number tracks**, **Base 10 equipment** and **100 bead strings** to support their developing understanding of addition using augmentation. Using their developing understanding of place value, they will move on to be able to use **Base 10 equipment** to make teens numbers using separate tens and ones.*

*Teachers demonstrate the use of **concrete equipment** and **number lines** to support calculations to **count on** in ones and children begin to use them independently.*

- ✓ **Number sentences** are always recorded.
- ✓ **Children add on above the number track and number line and subtract underneath.**

E.g. when adding 11 and 5, they can make the 11 using a ten rod and a unit. The equipment is set out at this stage horizontally to support children's understanding of number tracks and number lines.



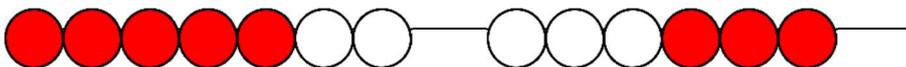
The ones can then be combined to aid with seeing the final total, e.g.  $11 + 5 = 16$ .



If possible, they should use two different colours of base 10 equipment so that the initial amounts can still be seen.

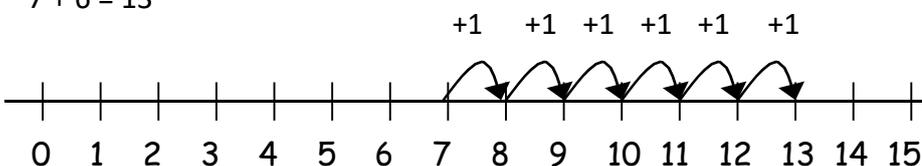
**Bead strings** or **bead bars** can be used to model addition including bridging through ten by counting on 3 then counting on 3.

- ✓ **Number sentences** are always recorded.



**Number lines** and **practical resources** will help to support addition calculations. Teachers demonstrate the use of the number line.

$$7 + 6 = 13$$



## End of Y2 objectives

Solve problems with addition and subtraction:

Using concrete objects and pictorial representations, including those involving numbers, quantities and measures applying their increasing knowledge of mental and written methods

Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100

Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:

- a two-digit number and ones
- a two-digit number and tens
- two two-digit numbers
- adding three one-digit numbers

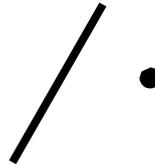
Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot

Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.

**Concrete** – Base 10 equipment

**Pictorial representation** – empty number line

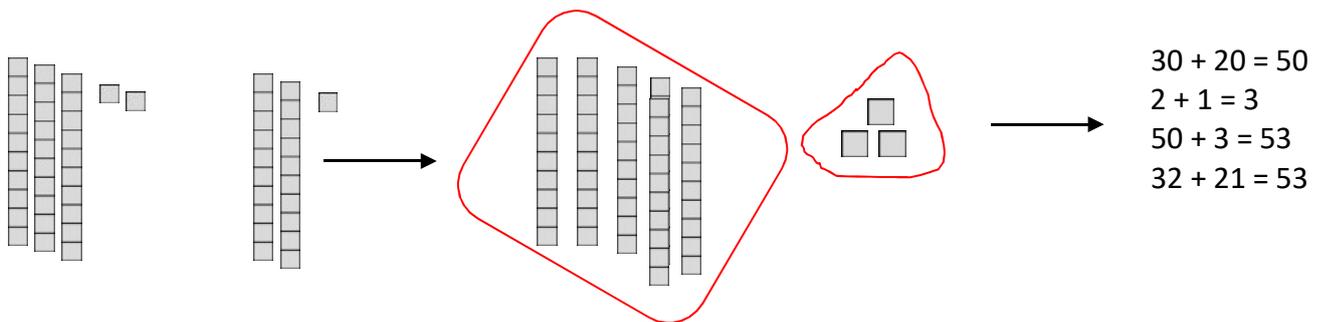
**Symbolic** – written number sentence



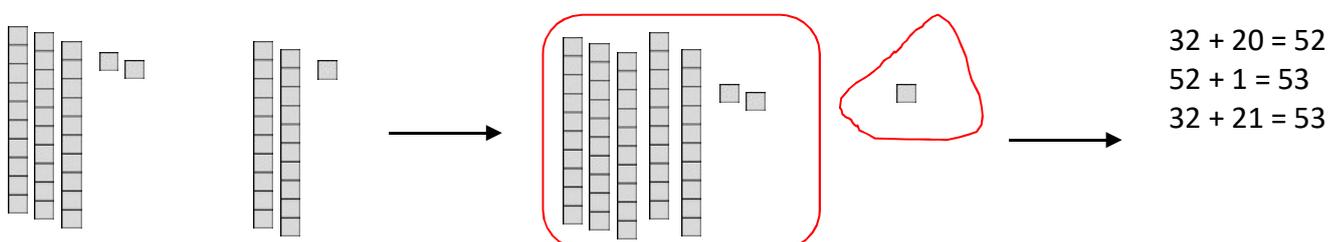
Children will continue to practically use the **Base 10 equipment** to support their understanding of addition starting with the larger number and counting on (**augmentation**) which will lead to developing mental strategies. This will be represented by the recording of the 'empty number lines' with **Base 10 equipment** used practically alongside e.g. **DO – TALK – RECORD**.

- ✓ **Number sentences** are always recorded.
- ✓ **Children add on above the empty number line and subtract underneath.**

**Step 1** - Children will continue to use the **Base 10 equipment** to support their calculations. For example, to calculate  $32 + 21$ , they can make the individual amounts; counting the tens first and then count on the ones (**partition and recombine**).



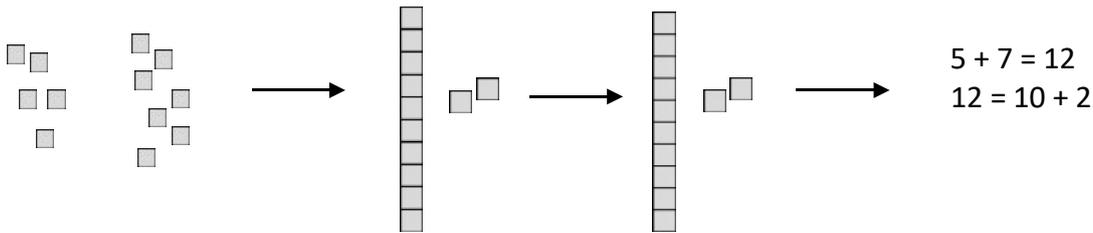
**Step 2** - Children will be encouraged to develop a more efficient strategy of only partitioning the smallest number, then adding the tens and then ones to the largest number to find the total (**augmentation – counting on**). This helps to support children in adding ten and later multiples of ten to two digit numbers. For example to calculate  $32 + 21$ :



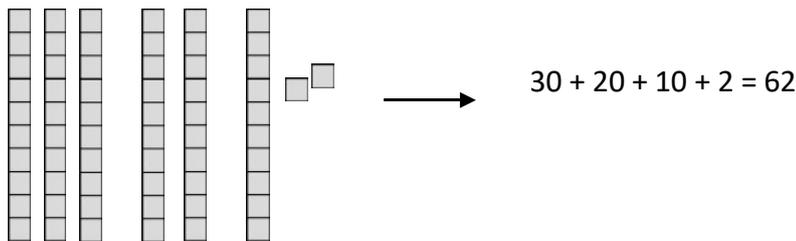
Step 3 - When the ones total more than 10, children should be encouraged to exchange 10 ones for 1 ten. This is the start of children understanding 'carrying' in vertical addition. For example, when calculating  $35 + 27$ , they can represent the amounts using Base 10 as shown:



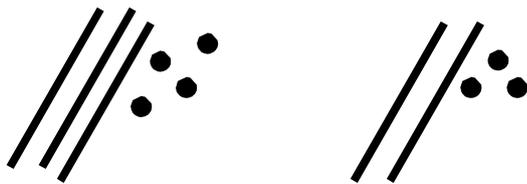
Then, identifying the fact that there are enough ones to exchange for a ten, they can carry out this exchange:



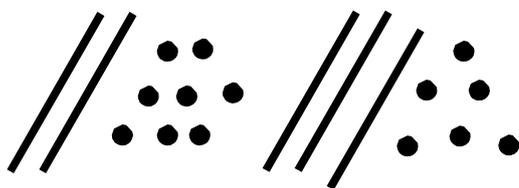
To leave:



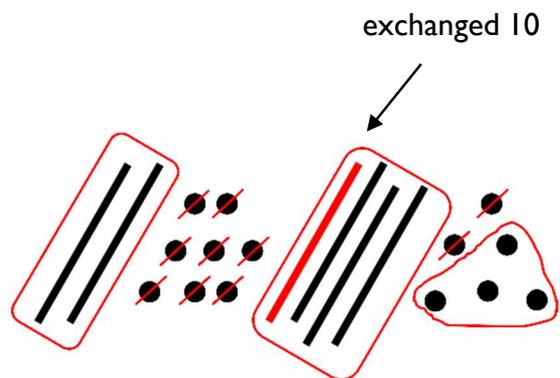
Children can also record the calculations using their own drawings of the Base 10 equipment (as slanted lines for the 10 rods and dots for the unit blocks) e.g.  $34 + 23 =$



With exchange: e.g.  $28 + 36 =$



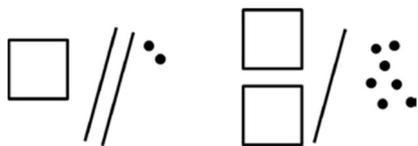
will become



so  $28 + 36 = 64$

It is important that children circle the remaining tens and ones after exchange to identify the amount remaining.

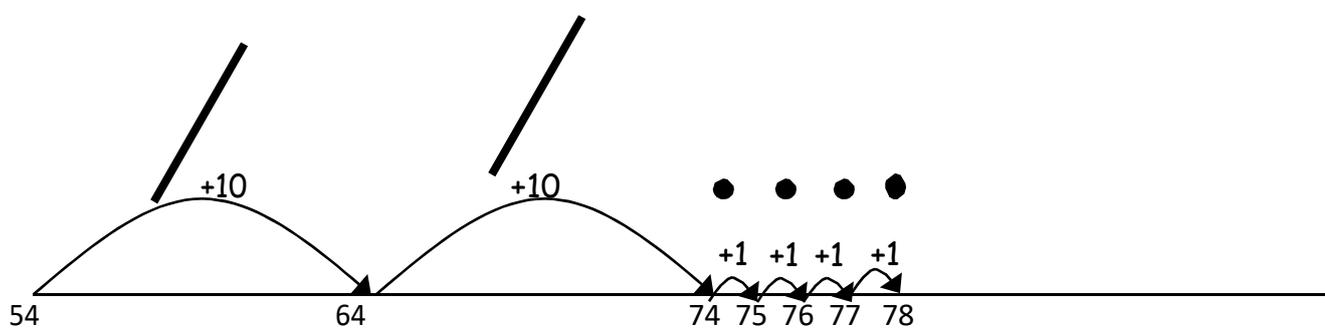
This method can also be used with adding three digit numbers, e.g. 122 + 217 using a square as the representation of 100.



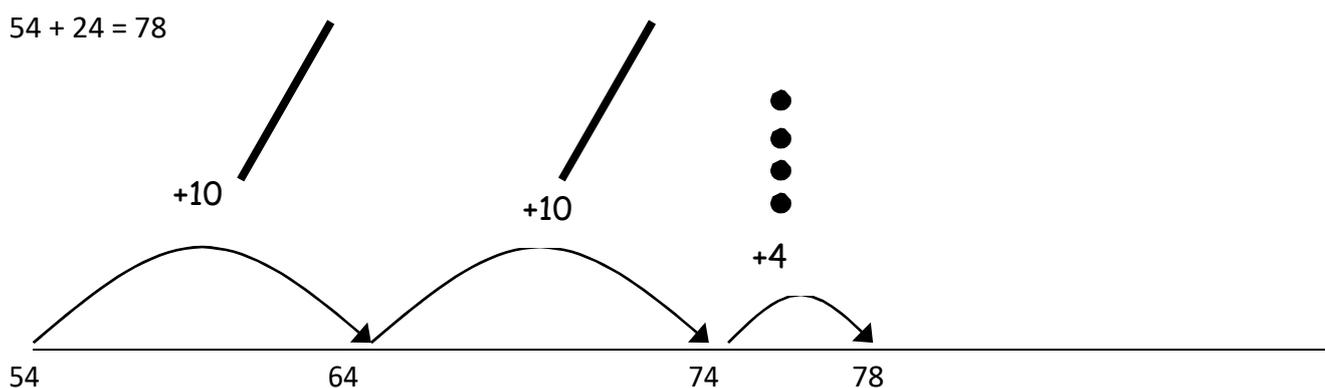
**Consider how each child is able to calculate the questions e.g. using concrete objects, pictorial representations or mentally. This will help to support differentiation but also be important information to pass onto the Y3 teacher. Children should gradually develop in confidence and no longer need the concrete equipment or the pictorial representation. Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.**

- ✓ **Number sentences** are always recorded.
- ✓ **Children add on above the empty number line and subtract underneath.**

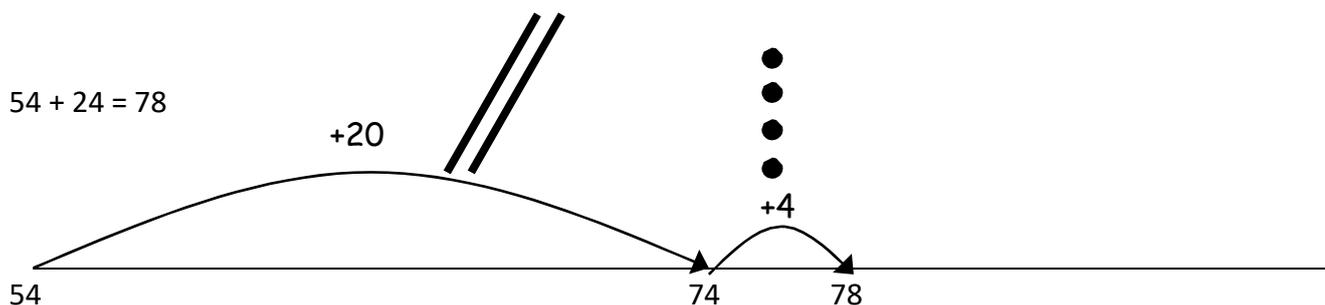
- ✓ **First counting on in tens and ones.**  $54 + 24 = 78$



- ✓ **Then helping children to become more efficient by adding the ones in one jump (by using the known fact  $4 + 4 = 8$ ).**

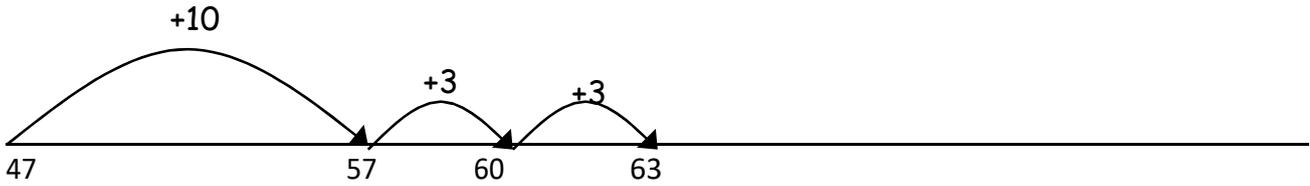


- ✓ **Followed by adding the tens in one jump and the ones in one jump.**



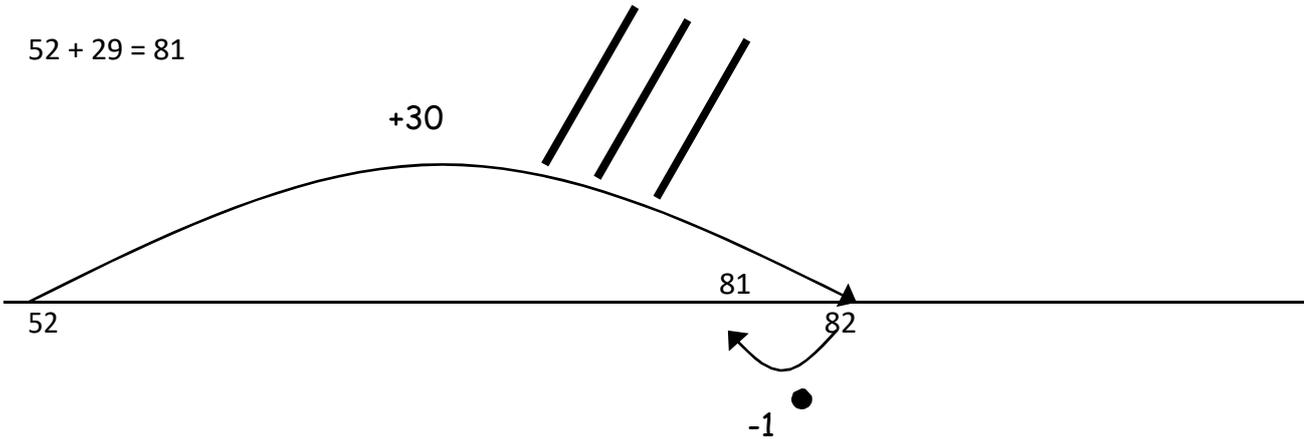
✓ **Bridging through ten can help children become more efficient.**

$$47 + 16 = 63$$



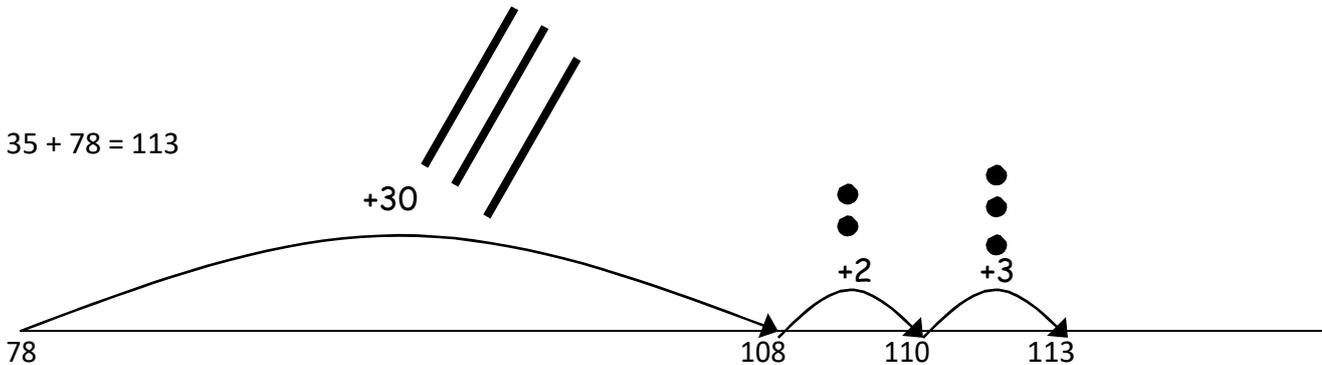
✓ **Rounding and adjusting by ten/s can help children become more efficient.**

$$52 + 29 = 81$$



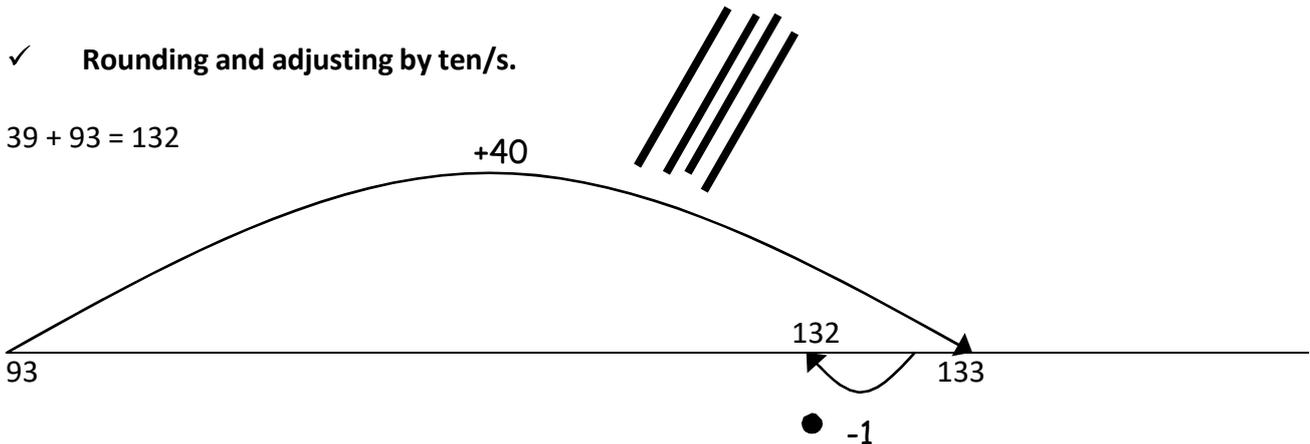
Children will continue to use empty number lines with increasingly large numbers, including rounding and adjusting if appropriate.

✓ **Count on from the largest number irrespective of the order of the calculation.**



✓ **Rounding and adjusting by ten/s.**

$$39 + 93 = 132$$



## **End of Y3 Objectives**

**Add and subtract numbers mentally, including:**

- a three-digit number and ones
- a three-digit number and tens
- a three-digit number and hundreds

**Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.**

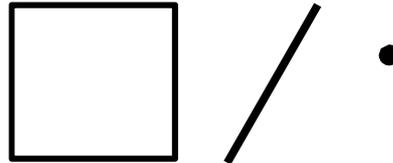
**Estimate the answer to a calculation and use inverse operations to check answers.**

**Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.**

**Concrete** – Base 10 equipment

**Pictorial representation** – empty number line

**Symbolic** – written number sentences



They may use their own **drawings** of the **Base 10 equipment** (square for 100 block, lines for the 10 rods and dots for the unit blocks) for **recording calculations**.

*\*Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4*

*It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.*

**THERE ARE TWO MAIN ASPECTS OF CALCULATIONS IN Y3 – DEVELOPING MENTAL STRATEGIES AND DEVELOPING FORMAL WRITTEN METHODS.**

### **Developing mental strategies**

**Add and subtract numbers mentally, including: a three-digit number and ones e.g.**

240 + 5 (adding from a multiple of 10)

345 + 7 (crossing the tens boundary)

898 + 6 (crossing the hundreds boundary)

986 + 8 (crossing the thousands boundary)

678 + 9 (add 10 and adjust by 1)

**Add and subtract numbers mentally, including: a three-digit number and tens e.g.**

240 + 10, 390 + 10 and 990 and 10 (using multiples of 10)

243 + 10 (crossing tens boundary), 397 + 10 (crossing hundreds boundary) and 996 + 10 (crossing thousands boundary) – all starting numbers that are not multiples of 10

**Add and subtract numbers mentally, including: a three-digit number and hundreds e.g.**

500 + 100, 900 + 100, 300 + 400 and 600 + 500 (crossing thousands boundary) - all starting numbers that are multiples of 100

452 + 200, 779 + 600, 599 + 600 (round and adjust) - all starting numbers that are not multiples of 100

To support the addition of numbers **mentally** in Y3, children should build upon their Y2 knowledge and experience (see previous Y2 guidance).

Children will continue to practically use the **Base 10 equipment** alongside the **pictorial representation of an empty number line** e.g. **DO – TALK – RECORD**. Children will continue to use empty number lines with increasingly large numbers, including starting with the larger number and counting on (augmentation), rounding and adjusting if appropriate. **The pictorial recording as an empty number line will build upon Y2 examples (see previous Y2 guidance)**

Children will gradually develop in confidence and no longer need the **concrete equipment** or the **pictorial representation of an empty number line**. Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies. Children should always look for the most efficient way to calculate and always consider the place value of the digits involved.

- ✓ **Number sentences** are always recorded.
- ✓ **Children add on above the empty number line and subtract underneath.**

### **Developing formal written methods**

Children will build on their knowledge of using **Base 10 equipment** from Y2 and continue to use this to support the transition into a formal written method of columnar addition for adding numbers up to three digits. The **Base 10 equipment** should be practically used and the **formal written methods of columnar addition** be recorded by the children. Clear modelling and explanation by the teacher and opportunity to **DO – TALK – RECORD** by the children is imperative.

Children will begin to explore the **formal written methods of columnar addition** with two –digit calculations. It is very important that children can confidently **mentally calculate the Y2 requirements** linked to adding (two-digit number and ones, a two-digit number and tens, two two-digit numbers and adding three one-digit numbers). The introduction of the **the formal written methods of columnar addition** for two digit numbers is to support children in learning a new method/process with numbers that they are confident in before moving onto three digit numbers.

Progression of formal written methods of columnar addition in Y3-

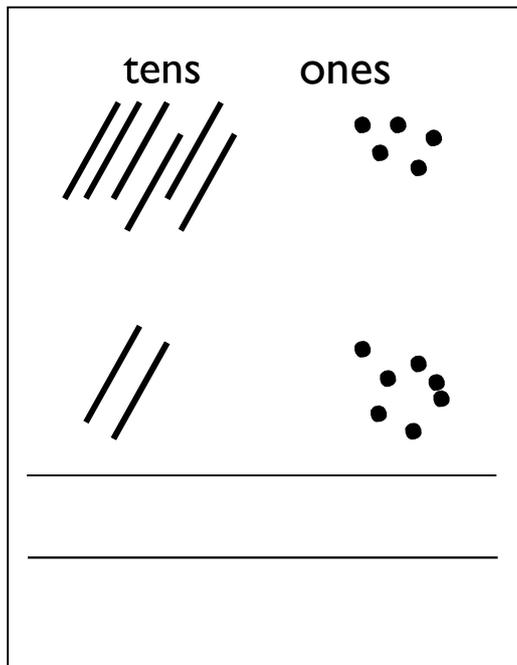
- Add 2 numbers with two digits together without exchange between the ones and tens.
- Add 2 numbers with two digits together with exchange.
- Add 2 numbers with three digits together without exchange between the ones and tens.
- Add 2 numbers with three digits together with exchange.

Children will build on their knowledge of using Base 10 equipment from Y2 and continue to use the idea of exchange.

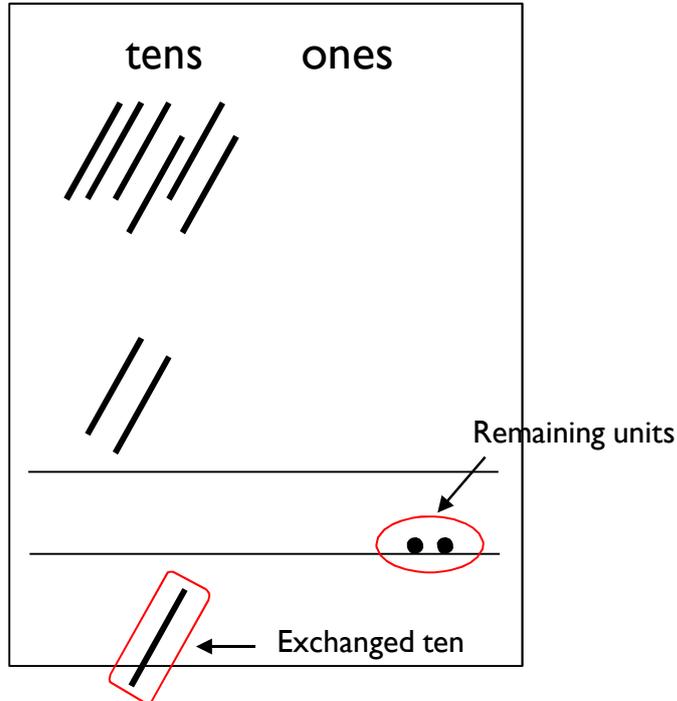
Children should add the **least significant digits** first (i.e. start with the ones), and in an identical method to that from year 2, should identify whether there are greater than ten ones which can be exchanged for one ten.

They can use a place value grid to begin to set the calculation out vertically and to support their knowledge of exchange between columns (as in Step 1 in the diagram below).e.g.  $65 + 27$

Step 1



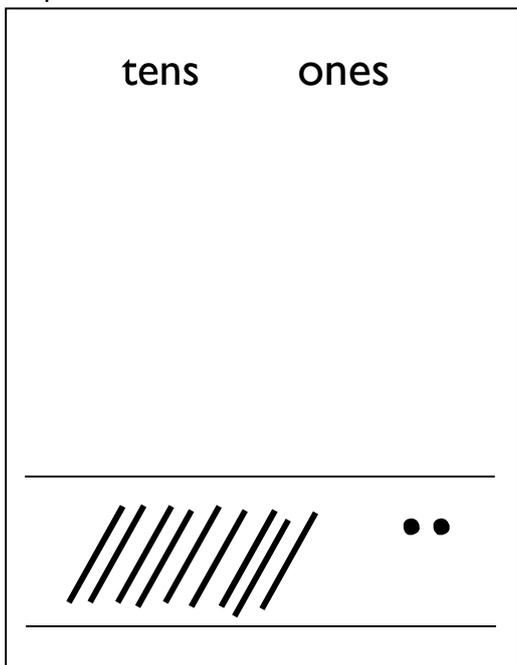
Step 2



Children would exchange ten ones for a ten, placing the exchanged ten below the equals sign. Any remaining ones that cannot be exchanged for a ten move into the equals sign as they are the ones part of the answer (as in the diagram in Step 2 above).

If there are any tens that can be exchanged for a hundred, this can be done next. If not, the tens move into the equals sign as they are the tens part of the answer (as in the diagram in Step 3 below).

Step 3



**Written method**

Step 1	Step 2	Step 3																																																						
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Children should utilise this practical method to link their understanding of exchange to how the column method is set out. Teachers should model the written method alongside this practical method initially. This should progress to children utilising the written and practical methods alongside each other and finally, and when they are ready, to children utilising just the written method. By the end of year 3, children should also extend this method for three digit numbers – please see guidance for Y4.

### End of Y4 objectives

**Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate.**

**Estimate and use inverse operations to check answers to a calculation.**

**Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.**

Children will build on their knowledge of using **Base 10 equipment** from Y3 and continue to use this to support the transition into a formal written method of columnar addition for adding numbers up to four digits. The **Base 10 equipment** should be practically used and the **formal written methods of columnar addition** be recorded by the children. Clear modelling and explanation by the teacher and opportunity to **DO – TALK – RECORD** by the children is imperative.

Children will gradually develop in confidence and no longer need the **concrete equipment** because they fully understand the quantity value of the digits and are confident with working through and recording the **formal written methods of columnar addition**. Children can now start to fully explore carrying below the line for all columns.

Children will move to Y4 using whichever method they were using as they transitioned from Y3.

Progression of formal written methods of columnar addition in Y4-

- Ensure children are fully confident in methods set out in Y3.
- Add three numbers with three digits with exchange.
- Add 2 numbers with four digits together without exchange between the ones and tens.
- Add 2 numbers with four digits together with exchange.
- Add three numbers with four digits together without exchange between the ones and tens.

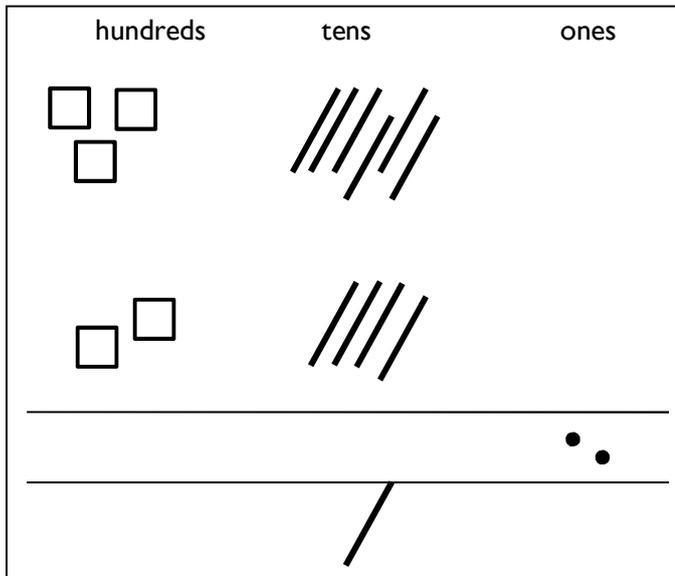


#### Step 1

hundreds	tens	ones
<hr/>		
<hr/>		

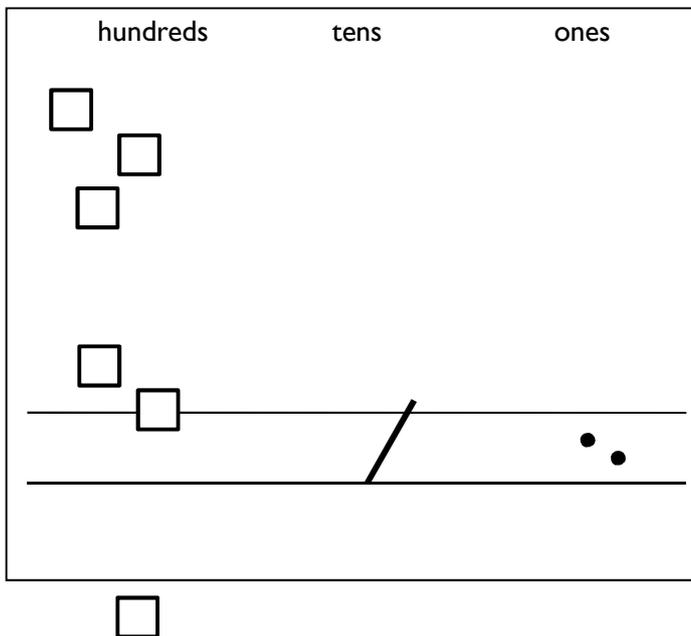
$$\begin{array}{r} \text{H} \quad \text{T} \quad \text{U} \\ 3 \quad 6 \quad 5 \\ + 2 \quad 4 \quad 7 \\ \hline \\ \hline \end{array}$$

Step 2



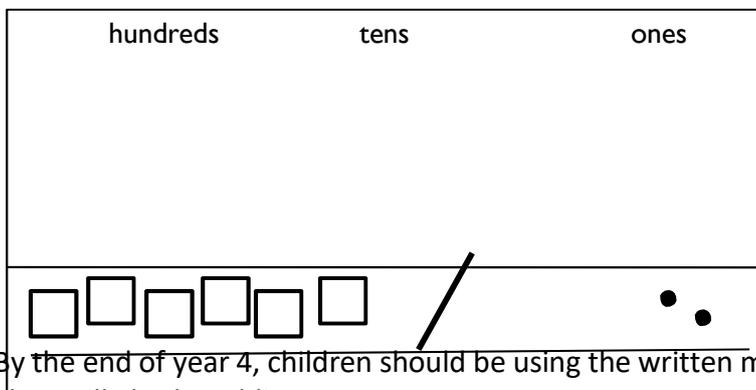
$$\begin{array}{r}
 \text{H} \quad \text{T} \quad \text{U} \\
 3 \quad 6 \quad 5 \\
 + 2 \quad 4 \quad 7 \\
 \hline
 \phantom{0} \quad \phantom{0} \quad 2 \\
 \hline
 \phantom{0} \quad | \phantom{0}
 \end{array}$$

Step 3



$$\begin{array}{r}
 \text{H} \quad \text{T} \quad \text{U} \\
 3 \quad 6 \quad 5 \\
 + 2 \quad 4 \quad 7 \\
 \hline
 \phantom{0} \quad | \quad 2 \\
 \hline
 \phantom{0} \quad | \quad |
 \end{array}$$

Step 4



$$\begin{array}{r}
 \text{H} \quad \text{T} \quad \text{U} \\
 3 \quad 6 \quad 5 \\
 + 2 \quad 4 \quad 7 \\
 \hline
 6 \quad | \quad 2 \\
 \hline
 \phantom{0} \quad | \quad |
 \end{array}$$

By the end of year 4, children should be using the written method confidently and with understanding. They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- *decimals with one decimal place, knowing that the decimal points line up under one another.*

### End of Y5 Objectives

Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction).

Add and subtract numbers mentally with increasingly large numbers.

Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.

Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Children should continue to use the carrying method to solve calculations such as:

$$\begin{array}{r} 3364 \\ + 247 \\ \hline 3611 \\ \hline \end{array}$$

$$\begin{array}{r} 3121 \\ + 148 \\ \hline 3306 \\ \hline \end{array}$$

$$\begin{array}{r} 3.56 \\ + 2.47 \\ \hline 6.03 \\ \hline \end{array}$$

They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with each number having the same number of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another

### End of Y6 Objectives

Perform mental calculations, including with mixed operations and large numbers.

Use their knowledge of the order of operations to carry out calculations involving the four operations.

Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.

Solve problems involving addition, subtraction, multiplication and division.

Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.

Children should extend the carrying method and use it to add whole numbers and decimals with any number of digits.

$$\begin{array}{r} 42 \\ 6432 \\ 786 \\ + 4681 \\ \hline 11944 \\ \hline \end{array}$$

$$\begin{array}{r} 401.20 \\ + 26.85 \\ \hline 428.76 \\ \hline \end{array}$$

When adding decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20.

They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with mixed numbers of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another.

## PROGRESSION TOWARDS A WRITTEN METHOD FOR SUBTRACTION

In developing a written method for subtraction, it is important that children understand the concept of subtraction, in that it is:

- Removal of an amount from a larger group (take away)
- Comparison of two amounts (difference)
- Reducing linked to measurement e.g. time, length, capacity (reduce)
- Subtracting whole, decimal and negative numbers

Also, to understand and work with certain principles:

- the inverse of addition
- not commutative i.e.  $5 - 3$  is not the same as  $3 - 5$
- not associative i.e.  $10 - 3 - 2$  is not the same as  $10 - (3 - 2)$
- = sign means same value but 'looks different'

**Always ensure that children are provided with an opportunity to access**

- ✓ Concrete
- ✓ Pictorial representation
- ✓ Symbolic

**DO – TALK – RECORD**

### Early Learning Goal

**Children count reliably with numbers from one to 20, place them in order and say which number is one more or one less than a given number.**

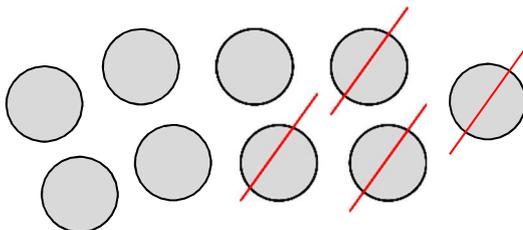
**Using quantities and objects, they add and subtract two single-digit numbers and count on and back to find the answer.**

**They solve problems including doubling, halving and sharing.**

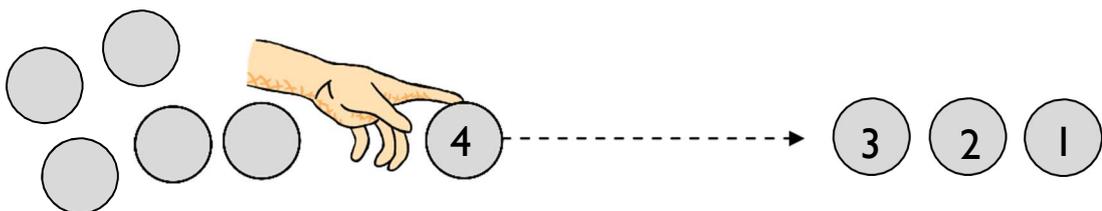
*Children should experience practical calculation opportunities using a wide variety of **practical** equipment, including small world play, role play, counters, cubes etc. They should be encouraged to develop ways of recording calculations using **pictures** and enhance the learning experience of developing a **mental picture** of the number system in their heads to use for calculation.*

### **Taking away**

Children will begin to develop their ability to subtract by using **practical equipment** to count out the first number and then remove or take away the second number to find the solution by counting how many are left e.g.  $9 - 4$ .



For illustration purposes, the amount being taken away are show crossed out. Children would be encouraged to physically remove these using touch counting.



By touch counting and dragging in this way, it allows children to keep track of how many they are removing so they do not have to keep recounting. They will then touch count the amount that are left to find the answer.

Those who are ready may record their own calculations e.g.  $8 - 5 = 3$ .



**Concrete** – counting animals  
**Pictorial representation** – see drawings  
**Symbolic** – written number sentence  
 Children **who are ready** may record this as:  
 $8 - 5 = 3$

**End of Y1 Objectives**  
 Read, write and interpret mathematical statements involving addition (+), subtraction (–) and equals (=) signs.  
 Represent and use number bonds and related subtraction facts within 20.  
 Add and subtract one-digit and two-digit numbers to 20, including zero using concrete and pictorial representations).  
 Solve one-step problems that involve addition and subtraction, using concrete objects and pictorial representations, and missing number problems such as  $7 = \square - 9$ .

*Children will continue to use practical equipment e.g. counters to take away. To become more efficient they will move on to the use of **number tracks**, **Base 10 equipment** and **bead strings of 20** to support their developing understanding of subtraction.*

*Teachers demonstrate the use of **concrete equipment** and **number lines** to support calculations to **count back** in ones and children begin to use them independently.*

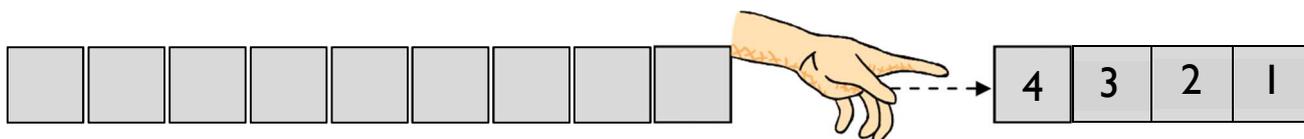
- ✓ **Number sentences** are always recorded.
- ✓ **Children add on above the number track and number line and subtract underneath.**

**Subtraction as taking away**

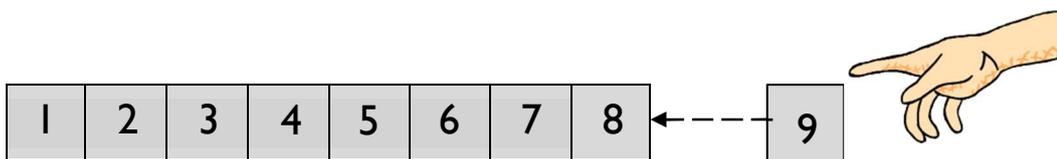
Children will continue to use practical equipment and taking away strategies (counting back). To avoid the need to exchange for subtraction at this stage, it is advisable to continue to use equipment such as counters, cubes and the ones from the Base 10 equipment, but not the tens, e.g.  $13 - 4$



Touch count and remove the number to be taken away, in this case 4.

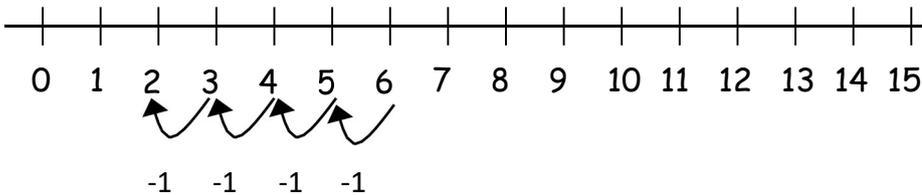


It is important that children keep track of how many have been removed. Touch count to find the number that remains and record number sentence  $13 - 4 = 9$ .

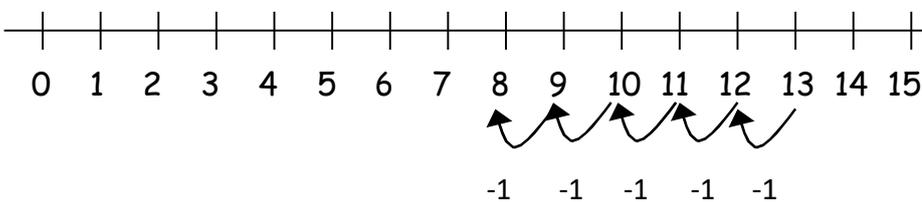


**Number lines** and **practical resources** will help to support subtraction calculations. Teachers demonstrate the use of the number line.

$$6 - 4 = 2$$

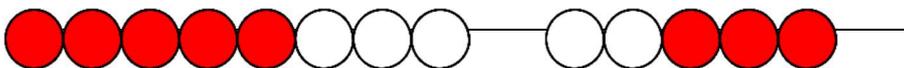


$$13 - 5 = 8$$



**Bead strings of 20** or **bead bars** can be used to illustrate subtraction bridging through ten by counting back 3 then counting back 2 for  $13 - 5 = 8$ .

✓ **Number sentences** are always recorded.



**Subtraction as finding the difference**

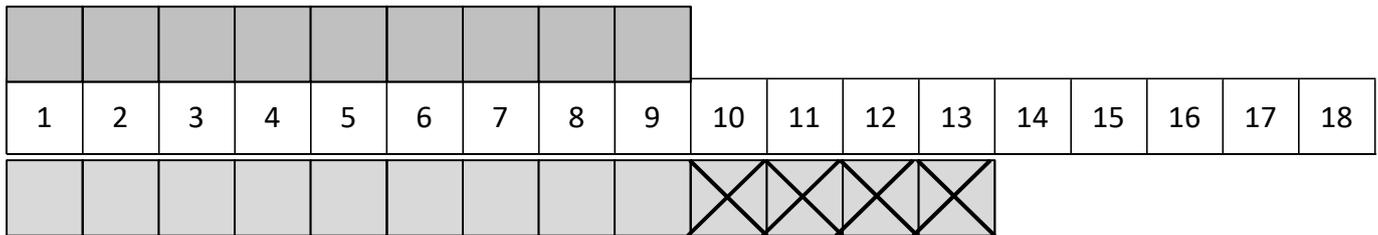
In preparation for understanding how to find the difference by counting up, children should be shown that finding the difference is linked to subtraction and the teacher should ensure the children know that it is an appropriate strategy to use when the numbers are close together.

e.g.  $13 - 9$

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	

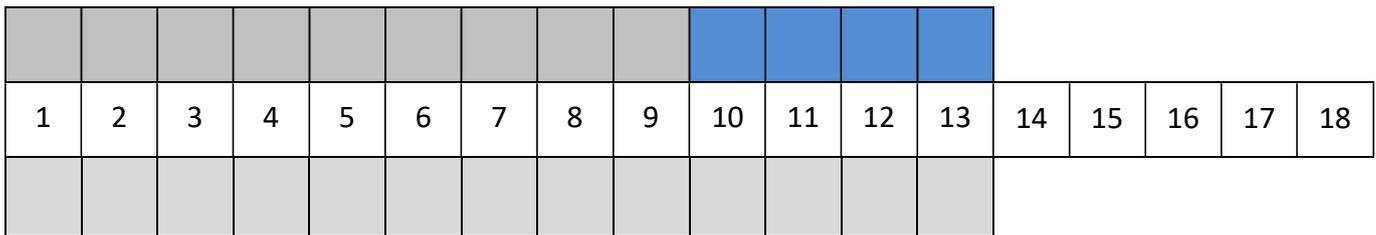
Children should use the Base 10 unit cubes and count out the correct amounts, placing one set above the number track and one below. To find the difference, children need to identify how to make the two amounts the same. This should begin by removing cubes from the larger amount, one at a time, until it

is the same size as the smaller amount. As each cube is removed the children count how many are being removed.



Children should understand that this calculation is  $13 - ? = 9$

The next stage of finding the difference regards making the smaller amount the same size as the larger amount by counting on. Children should understand that this calculation is  $9 + ? = 13$



***NB – It is useful to present the context of difference in real life contexts such as comparing two measurements or when interpreting block graphs.***

**End of Y2 objectives**

**Solve problems with addition and subtraction:**

**Using concrete objects and pictorial representations, including those involving numbers, quantities and measures applying their increasing knowledge of mental and written methods**

**Recall and use addition and subtraction facts to 20 fluently, and derive and use related facts up to 100**

**Add and subtract numbers using concrete objects, pictorial representations, and mentally, including:**

- a two-digit number and ones
- a two-digit number and tens
- two two-digit numbers
- adding three one-digit numbers

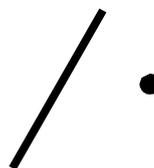
**Show that addition of two numbers can be done in any order (commutative) and subtraction of one number from another cannot**

**Recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems.**

**Concrete** – Base 10 equipment

**Pictorial representation** – empty number line

**Symbolic** – written number sentence

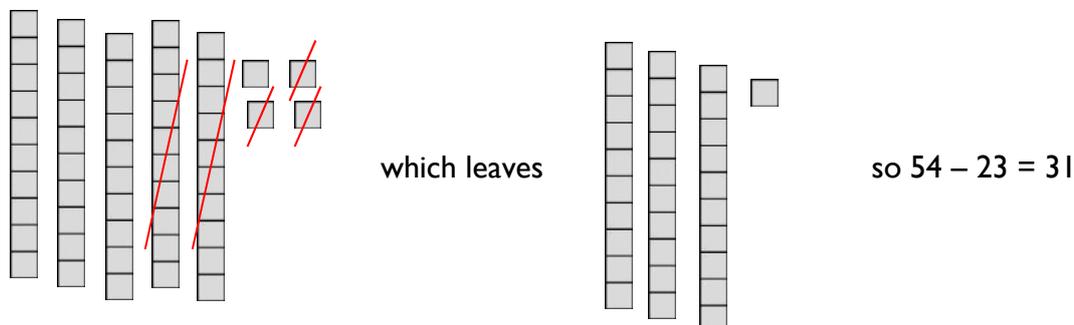


*Children will continue to practically use the **Base 10 equipment** to support their understanding of subtraction. This will be represented by the recording of the ‘empty number lines’ with Base 10 equipment used practically alongside e.g. **DO – TALK – RECORD.***

- ✓ **Number sentences** are always recorded.
- ✓ **Children add on above the empty number line and subtract underneath.**

### Subtraction as taking away

Children will begin to use the **Base 10 equipment** to support their calculations, still using a take away, or removal, method. They need to understand that the number being subtracted does not appear as an amount on its own, but rather as part of the larger amount. For example, to calculate  $54 - 23$ , children would count out 54 using the Base 10 equipment (5 tens and 4 ones). They need to consider whether there are enough ones to remove 3, in this case there are, so they would remove 3 ones and then two tens, counting up the answer of 3 tens and 1 unit to give 31.

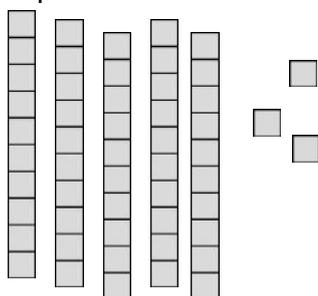


Children can also record the calculations using their own **drawings of the Base 10 equipment** (as slanted lines for the 10 rods and dots for the unit blocks), e.g. to calculate  $39 - 17$  children would draw 39 as 3 tens (lines) and 4 ones (dots) and would cross out 7 ones and then one ten, counting up the answer of 2 tens and 2 ones to give 22.

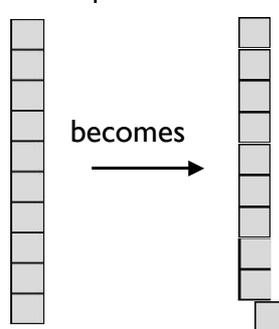


When the amount of ones to be subtracted is greater than the ones in the original number, an **exchange** method is required. This relies on children's understanding of ten ones being an equivalent amount to one ten. To calculate  $53 - 26$ , by using practical equipment, they would count out 53 using the tens and ones, as in Step 1. They need to consider whether there are enough ones/ones to remove 6. In this case there are not so they need to exchange a ten into ten ones to make sure that there are enough, as in step 2.

Step 1

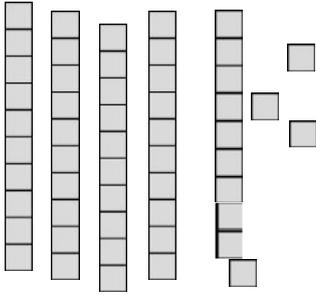


Step 2

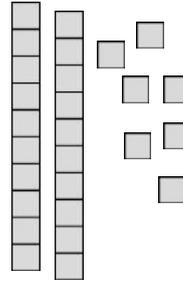


The children can now see the 53 represented as 40 and 13, still the same total, but partitioned in a different way, as in step 3 and can go on to take away the 26 from the calculation to leave 27 remaining, as in Step 4.

Step 3

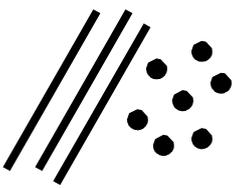


Step 4

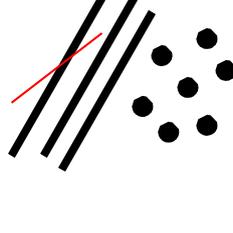


When recording their own drawings, when calculating  $37 - 19$ , children would cross out a ten and exchange for ten ones. Drawing them in a vertical line, as in Step 2, ensures that children create ten ones and do not get them confused with the ones that were already in place.

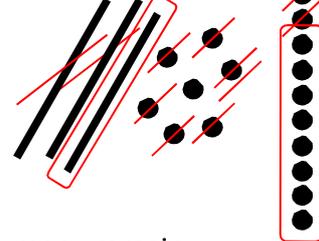
Step 1



Step 2



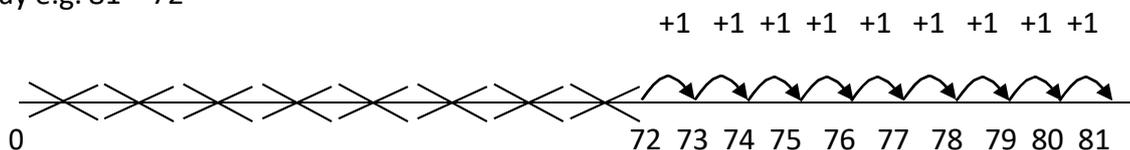
Step 3



Circling the tens and ones that remain will help children to identify how many remain.

### Subtraction as finding the difference

If the numbers involved in the calculation are close together or near to multiples of 10, 100 etc., children should be encouraged to recognise that it is more efficient to find the difference by counting up using a number line to support the mental calculation. Initially, 0 should be included on the number line to demonstrate that this portion (from 0-72) has been removed, which is similar to the process of taking away e.g.  $81 - 72 =$

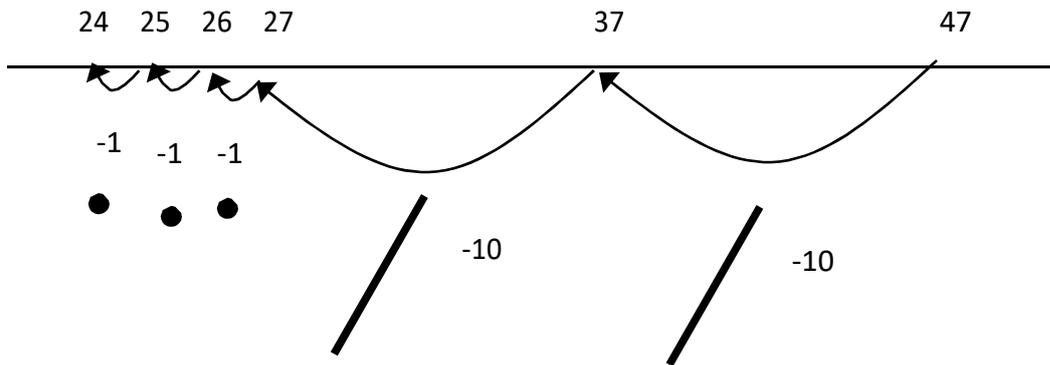


**Help children to become more efficient with counting on by:** subtracting the units in one jump.

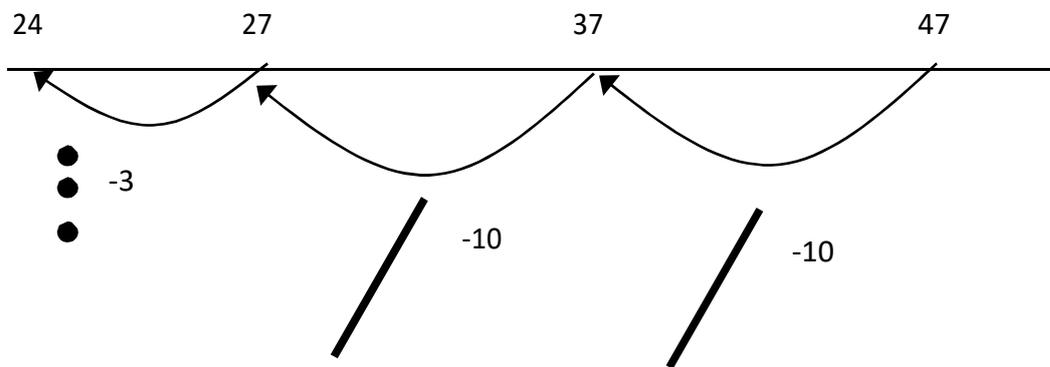
**Consider how each child is able to calculate the questions e.g. using concrete objects, pictorial representations or mentally. This will help to support differentiation but also be important information to pass onto the Y3 teacher. Children should gradually develop in confidence and no longer need the concrete equipment or the pictorial representation. Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies.**

- ✓ **Number sentences** are always recorded.
- ✓ **Children add on above the empty number line and subtract underneath**

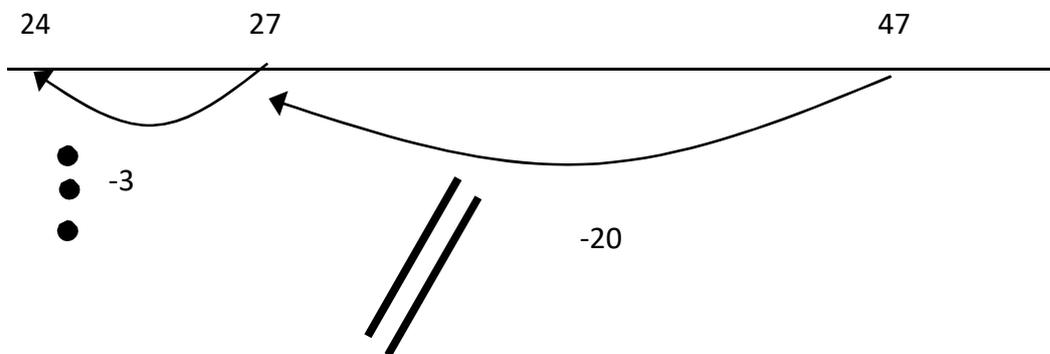
- ✓ **First counting back in tens and ones e.g.  $47 - 23 = 24$ .**



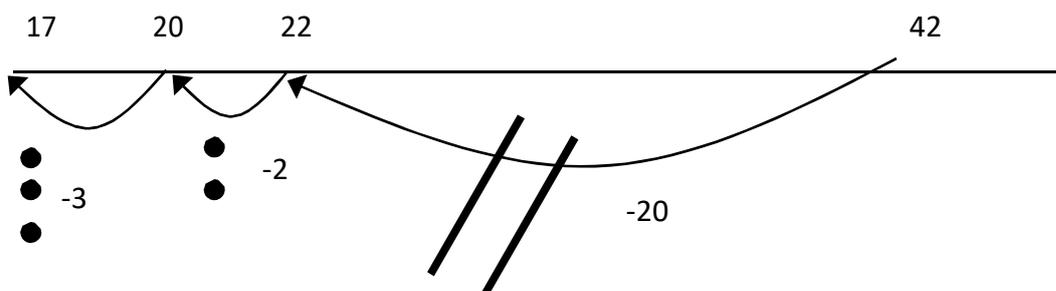
- ✓ **Then helping children to become more efficient by subtracting the ones in one jump (by using the known fact  $7 - 3 = 4$ ) e.g.  $47 - 23 = 24$**



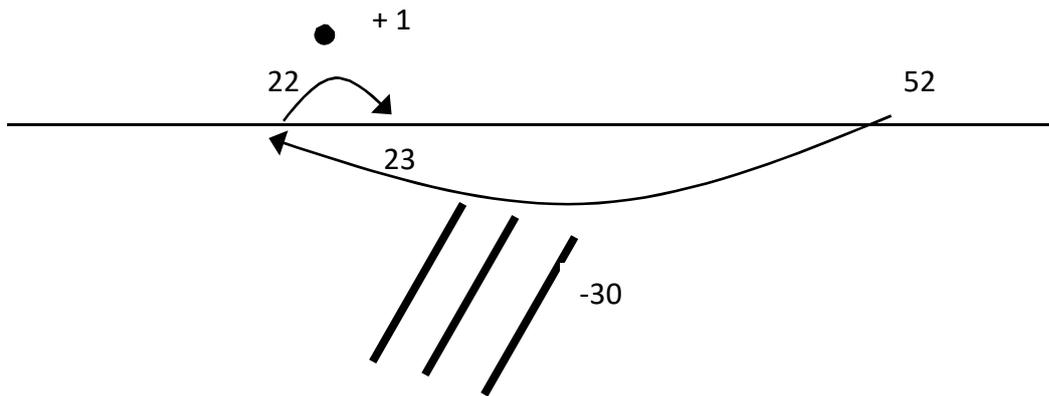
- ✓ **Followed by subtracting the tens in one jump and the ones in one jump e.g.  $47 - 23 = 24$ .**



- ✓ **Bridging through ten can help children become more efficient e.g.  $42 - 25 = 17$ .**



- ✓ Rounding and adjusting by ten/s can help children become more efficient e.g.  $52 - 29 = 23$ .



### **End of Y3 Objectives**

**Add and subtract numbers mentally, including:**

- a three-digit number and ones
- a three-digit number and tens
- a three-digit number and hundreds

**Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction.**

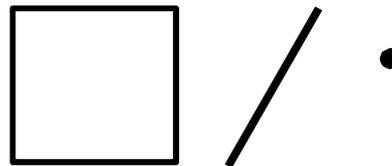
**Estimate the answer to a calculation and use inverse operations to check answers.**

**Solve problems, including missing number problems, using number facts, place value, and more complex addition and subtraction.**

**Concrete** – Base 10 equipment

**Pictorial representation** – empty number line

**Symbolic** – written number sentences



They may use their own **drawings** of the **Base 10 equipment** (square for 100 block, lines for the 10 rods and dots for the unit blocks) for **recording calculations**.

*\*Although the objective suggests that children should be using formal written methods, the National Curriculum document states “The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study.” p4*

*It is more beneficial for children’s understanding to go through the expanded methods of calculation as steps of development towards a formal written method.*

**THERE ARE TWO MAIN ASPECTS OF CALCULATIONS IN Y3 – DEVELOPING MENTAL STRATEGIES AND DEVELOPING FORMAL WRITTEN METHODS.**

### **Developing mental strategies**

**Add and subtract numbers mentally, including: a three-digit number and ones e.g.**

240 - 5 (subtracting from a multiple of 10)

345 - 7 (crossing the tens boundary)

903 - 6 (crossing the hundreds boundary)

678 - 9 (take away 10 and adjust by 1)

**Add and subtract numbers mentally, including: a three-digit number and tens e.g.**

240 - 10, 390 - 10 (using multiples of 10)

243 - 10 (crossing tens boundary), 404 - 10 (crossing hundreds boundary) and 1006 - 10 (crossing thousands boundary) – all starting numbers that are not multiples of 10

**Add and subtract numbers mentally, including: a three-digit number and hundreds e.g.**

500 - 100, 900 - 100, 300 - 400 and 600 - 500 (crossing thousands boundary) - all starting numbers that are multiples of 100

452 - 200, 779 - 600, 699 - 600 (round and adjust) - all starting numbers that are not multiples of 100

*To support the addition of numbers **mentally** in Y3, children should build upon their Y2 knowledge and experience (see previous Y2 guidance).*

*Children will continue to practically use the **Base 10 equipment** alongside the **pictorial representation** of an **empty number line** e.g. **DO – TALK – RECORD**. Children will continue to use empty number lines with increasingly large numbers, including taking away and finding the difference. **The pictorial recording as an empty number line will build upon Y2 examples (see previous Y2 guidance)***

*Children will gradually develop in confidence and no longer need the **concrete equipment** or the **pictorial representation** of an **empty number line**. Children will begin to use informal pencil and paper methods (jottings) to support, record and explain partial mental methods building on existing mental strategies. Children should always look for the most efficient way to calculate and always consider the place value of the digits involved.*

- ✓ **Number sentences** are always recorded.
- ✓ **Children add on above the empty number line and subtract underneath.**

**Developing formal written methods**

*Children will build on their knowledge of using **Base 10 equipment** from Y2 and continue to use this to support the transition into a formal written method of columnar addition for adding numbers up to three digits. The **Base 10 equipment** should be practically used and the **formal written methods of columnar subtraction** be recorded by the children. Clear modelling and explanation by the teacher and opportunity to **DO – TALK – RECORD** by the children is imperative.*

*Children will begin to explore the **formal written methods of columnar subtraction** with two –digit calculations. It is very important that children can confidently **mentally calculate the Y2 requirements** linked to subtracting (two-digit number and ones, a two-digit number and tens, two two-digit numbers and adding three one-digit numbers). The introduction of the **the formal written methods of columnar subtraction** for two digit numbers is support children in learning a new method/process with numbers that they are confident in before moving onto three digit numbers.*

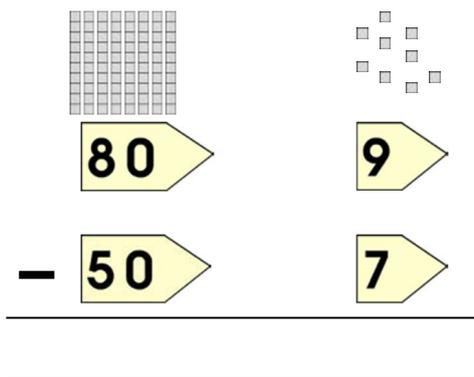
*Progression of formal written methods of columnar addition in Y3-*

- Subtract 2 numbers with two digits together without exchange between the ones and tens.
- Subtract 2 numbers with two digits together with exchange.
- Subtract 2 numbers with three digits together without exchange between the ones and tens.
- Subtract 2 numbers with three digits together with exchange.

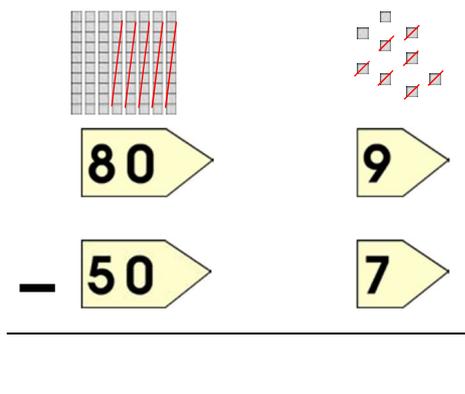
Children will build on their knowledge of using Base 10 equipment from Y2 and continue to use the idea of exchange. This process should be demonstrated using arrow cards to show the partitioning and Base

10 materials to represent the first number, removing the ones and tens as appropriate (as with the more informal method in Y2).

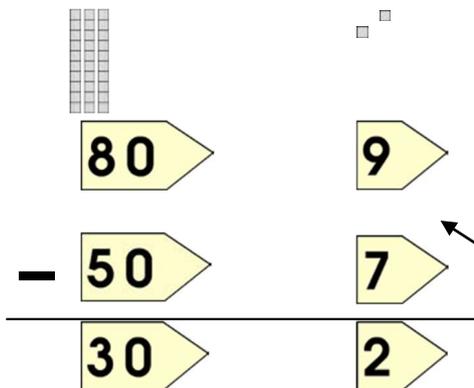
Step 1



Step 2



Step 3



*Emphasise that the second (bottom) number is being subtracted from the first (top) number rather than the lesser number from the greater. The calculation should be read 9 subtract 7.*

This will be recorded by the children as:

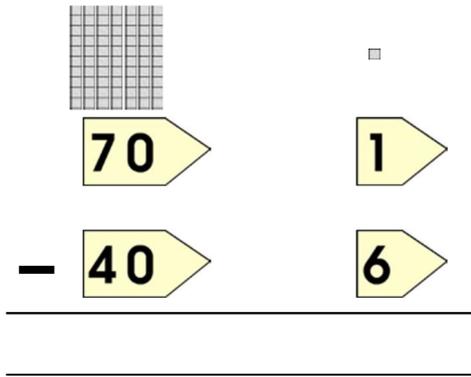
$$\begin{array}{r} 80 \rightarrow 9 \\ - 50 \rightarrow 7 \\ \hline 30 \rightarrow 2 \end{array} = 32$$

Children can also use jottings of the Base 10 materials (as in Year 2) to support with their calculation, as in the example below.

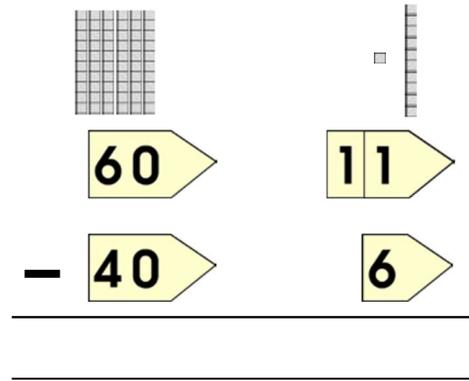
$$\begin{array}{r} 80 \rightarrow 9 \\ - 50 \rightarrow 7 \\ \hline 30 \rightarrow 2 \end{array} = 32$$

From this the children will begin to solve problems which involve exchange. Children need to consider whether there are enough ones/ones to remove 6. In this case there are not (Step 1) so they need to exchange a ten into ten ones to make sure that there are enough, as they have been doing in the method for Year 2 (Step 2). They should be able to see that the number is just partitioned in a different way, but the amount remains the same ( $71 = 70 + 1 = 60 + 11$ ).

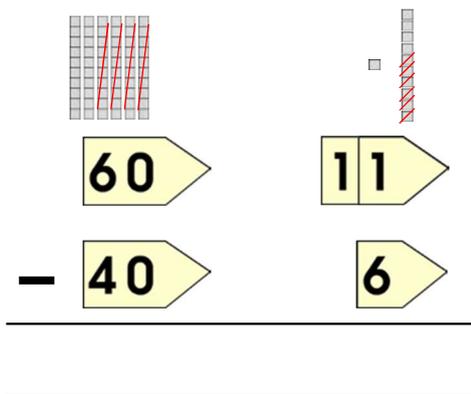
Step 1



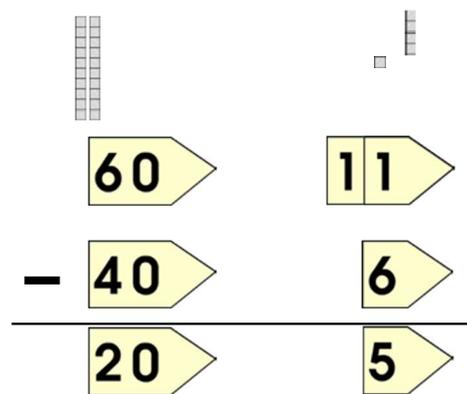
Step 2



Step 3



Step 4



This will be recorded by the children as:

$$\begin{array}{r}
 60 \\
 \cancel{70} \rightarrow 11 \\
 - 40 \rightarrow 6 \\
 \hline
 20 \rightarrow 5 = 25
 \end{array}$$

By the end of Y3, children should also extend this method for three digit numbers.

**End of Y4 objectives**

**Add and subtract numbers with up to 4 digits using the formal written methods of columnar addition and subtraction where appropriate.**

**Estimate and use inverse operations to check answers to a calculation.**

**Solve addition and subtraction two-step problems in contexts, deciding which operations and methods to use and why.**

Children will build on their knowledge of using **Base 10 equipment** from Y3 and continue to use this to support the transition into a formal written method of columnar subtraction for subtracting numbers up to four digits. The **Base 10 equipment** should be practically used and the **formal written methods of columnar subtraction** be recorded by the children. Clear modelling and explanation by the teacher and opportunity to **DO – TALK – RECORD** by the children is imperative.

Children will gradually develop in confidence and no longer need the **concrete equipment** because they fully understand the quantity value of the digits and are confident with working through and recording the **formal written methods of columnar subtraction**. Children can now start to fully explore carrying

below the line for all columns.

Children will move to Y4 using whichever method they were using as they transitioned from Y3.

Progression of formal written methods of columnar addition in Y4-

- Ensure children are fully confident in methods set out in Y3.
- Subtract 2 numbers with four digits together without exchange between the ones and tens.
- Subtract 2 numbers with four digits together with exchange.
- Subtract three numbers with four digits together without exchange between the ones and tens.



Children will move to Y4 using whichever method they were using as they transitioned from Y3.

Step 1

$$\begin{array}{r} 700 \rightarrow 50 \rightarrow 4 \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline \end{array}$$

Step 2 (exchanging from tens to ones)

$$\begin{array}{r} 700 \rightarrow \overset{40}{\cancel{50}} \rightarrow \overset{1}{4} \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline \end{array}$$

Step 3 (exchanging from hundreds to tens)

$$\begin{array}{r} \overset{600}{\cancel{700}} \rightarrow \overset{140}{\cancel{50}} \rightarrow \overset{1}{4} \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline \end{array}$$

Step 4

$$\begin{array}{r} \overset{600}{\cancel{700}} \rightarrow \overset{140}{\cancel{50}} \rightarrow \overset{1}{4} \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline 400 \rightarrow 60 \rightarrow 8 = 468 \end{array}$$

This would be recorded by the children as:

$$\begin{array}{r} \overset{600}{\cancel{700}} \rightarrow \overset{140}{\cancel{50}} \rightarrow \overset{1}{4} \\ - 200 \rightarrow 80 \rightarrow 6 \\ \hline 400 \rightarrow 60 \rightarrow 8 = 468 \end{array}$$

When children are ready, this leads on to the compact method of decomposition:

$$\begin{array}{r} \overset{6}{4} \overset{14}{\cancel{7}} \overset{1}{\cancel{5}} \overset{1}{4} \\ - 3 \quad 2 \quad 8 \quad 6 \\ \hline 1 \quad 4 \quad 6 \quad 8 \end{array}$$

By the end of Y4, children should be using the written method confidently and with understanding. They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals with one decimal place, knowing that the decimal points line up under one another.*

### **End of Y5 Objectives**

**Add and subtract whole numbers with more than 4 digits, including using formal written methods (columnar addition and subtraction).**

**Add and subtract numbers mentally with increasingly large numbers.**

**Use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.**

**Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.**

Children should continue to use the decomposition method to solve calculations such as:

$$\begin{array}{r} \overset{6}{\cancel{7}} \overset{6}{\cancel{10}} \overset{6}{\cancel{7}} \overset{12}{\cancel{12}} \\ - \quad 3 \quad 2 \quad 2 \quad 6 \\ \hline 3 \quad 8 \quad 4 \quad 6 \end{array}$$

$$\begin{array}{r} \overset{2}{\cancel{3}} \overset{13}{\cancel{4}} \overset{12}{\cancel{12}} \\ - \quad 1 \quad . \quad 7 \quad 6 \\ \hline 1 \quad . \quad 6 \quad 6 \end{array}$$

They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with each number having the same number of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another

### **End of Y6 Objectives**

**Perform mental calculations, including with mixed operations and large numbers.**

**Use their knowledge of the order of operations to carry out calculations involving the four operations.**

**Solve addition and subtraction multi-step problems in contexts, deciding which operations and methods to use and why.**

**Solve problems involving addition, subtraction, multiplication and division.**

**Use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy.**

Children should extend the decomposition method and use it to subtract whole numbers and decimals with any number of digits.

$$\begin{array}{r} \overset{5}{\cancel{6}} \overset{13}{\cancel{4}} \overset{13}{\cancel{13}} \overset{2}{\cancel{2}} \\ - \quad 4 \quad 6 \quad 8 \quad 1 \\ \hline 1 \quad 7 \quad 5 \quad 1 \end{array}$$

$$\begin{array}{r} \overset{3}{\cancel{4}} \overset{6}{\cancel{11}} \overset{6}{\cancel{7}} \overset{11}{\cancel{2}} \overset{10}{\cancel{10}} \\ - \quad 3 \quad 4 \quad . \quad 7 \quad 1 \\ \hline 3 \quad 8 \quad 2 \quad . \quad 4 \quad 9 \end{array}$$

When subtracting decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20.

They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with mixed numbers of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another.

## PROGRESSION TOWARDS A WRITTEN METHOD FOR MULTIPLICATION

In developing a written method for multiplication, it is important that children understand the concept of multiplication, in that:

- it is repeated addition
- can be represented as an array

Also, to understand and work with certain principles:

- the inverse of division
- commutative i.e.  $5 \times 3$  is the same as  $3 \times 5$
- associative i.e.  $2 \times 3 \times 5$  is the same as  $2 \times (3 \times 5)$
- = sign means same value but 'looks different'
- Equals sign can be in different positions i.e.  $3 \times 5 = 15$  and  $15 = 3 \times 5$

**Always ensure that children are provided with an opportunity to access**

- ✓ **Concrete**
- ✓ **Pictorial representation**
- ✓ **Symbolic**

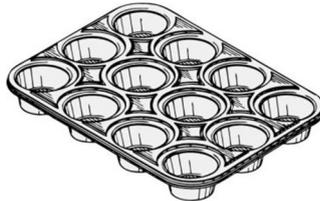
**DO – TALK – RECORD**

### **Early Learning Goal:**

**Children solve problems including doubling, halving and sharing.**

*Children are encouraged to develop a **mental picture** of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of **equipment**, including small world play, role play, counters, cubes etc.*

Children may also investigate putting items into resources such as egg boxes, ice cube trays and baking tins which are arrays.

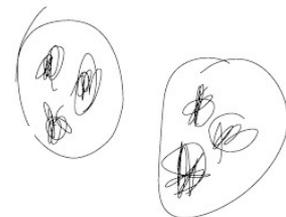


They may develop ways of recording calculations using pictures, etc.



A child's jotting showing the fingers on each hand as a double.

A child's jotting showing double three as three cookies on each plate.



### **End of Y1 Objectives:**

**Solve simple one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.**

In Y1, children will continue to solve multiplication problems using **practical equipment** and jottings. They may use the equipment to make groups of objects. Children should see everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc and use this in their learning, answering questions such as 'How many eggs would we need to fill the egg box? How do you know?'

**End of Y2 Objectives:**

Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.

Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication ( $\times$ ), division ( $\div$ ) and equals (=) signs.

Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.

Solve one-step problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

The following process of concrete, pictorial and symbolic should be explored every time children are introduced to a multiplication family.

Children should understand and be able to calculate multiplication as repeated addition, supported by:

**Concrete** – counters, cubes and 100 bead strings

**Pictorial representation** – number lines and empty number line

**Symbolic** – written number sentence

Children can discuss and record 2 multiplied by 5 in different ways:

$$2 + 2 + 2 + 2 + 2 = 10$$

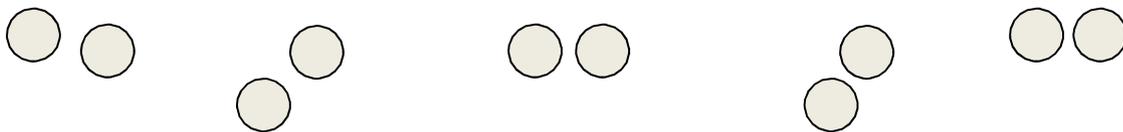
5 lots of 2

5 groups of 2

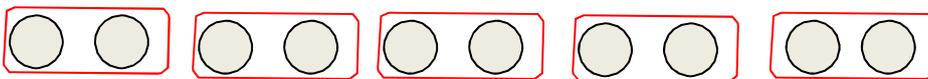
2 multiplied by 5

$$2 \times 5$$

2 multiplied by 5 can be shown as five groups of two with counters, either grouped in a random pattern, as below:

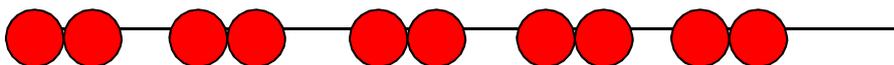


or in a more ordered pattern, with the groups of two indicated by the border outline:

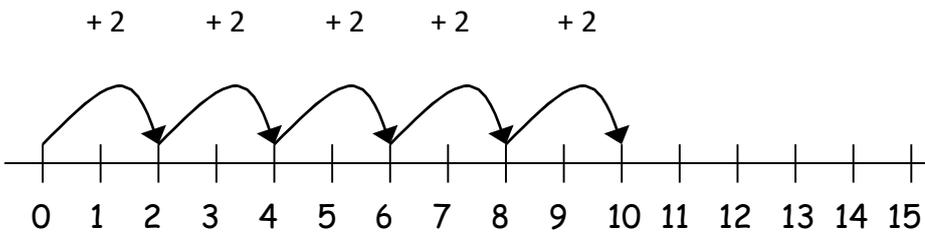


**100 Bead strings** or **bead bars** can be used to model repeated addition:

**Number sentences** are always recorded.

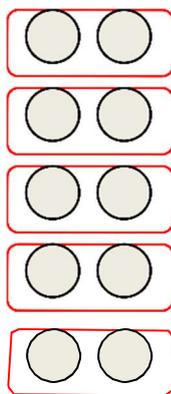


$$2 + 2 + 2 + 2 + 2 = 10$$



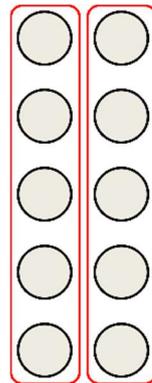
Children should then develop this knowledge to show how multiplication calculations can be represented by an array, (this knowledge will support with the development of the grid method in the future). Again, children should be encouraged to use **practical apparatus** and jottings to support their understanding, e.g.

$2 \times 5$  can be represented as an array in two forms (as it has commutativity):



$$2 + 2 + 2 + 2 + 2 = 10$$

$$2 \times 5 = 10$$



$$5 + 5 = 10$$

$$5 \times 2 = 10$$

Once children understand the commutative order of multiplication the order is irrelevant.

Partial tables/key facts box should be introduced every time children have learnt, understood and can recall multiplication facts. This method will also support doubling, halving, adding and subtracting.

1x	2
2x	4
4x	8
5x	10
10x	20
20x	40

Groups of 2  
Partial tables/key  
facts box

1x	5
2x	10
4x	20
5x	25
10x	50
20x	100

Groups of 5  
Partial tables/key  
facts box

1x	10
2x	20
4x	40
5x	50
10x	100
20x	200

Groups of 10  
Partial tables/key  
facts box

### End of Y3 Objectives

Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.

Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.

Solve problems, including missing number problems, involving multiplication and division, including integer scaling problems and correspondence problems in which  $n$  objects are connected to  $m$  objects.

*Please see guidance from Y2 and ensure children are confident and secure in multiplication objectives and understanding.*

*\*Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4. It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.*

The following process of concrete, pictorial and symbolic should be explored every children are introduced to a multiplication family e.g. 3, 4 and 8.

Children should understand and be able to calculate multiplication as repeated addition, supported by:

**Concrete** – counters, cubes and 100 bead strings

**Pictorial representation** – number lines and empty number line

**Symbolic** – written number sentence

Children can discuss and record 3 multiplied by 5 in different ways:

$$3 + 3 + 3 + 3 + 3 = 15$$

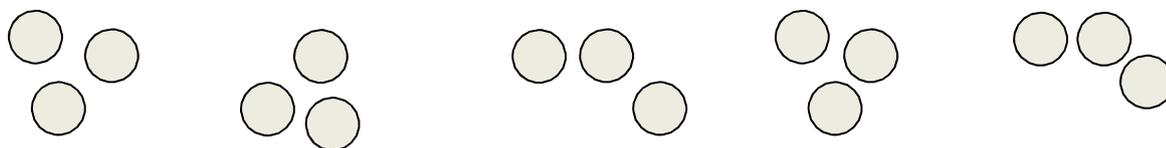
5 lots of 3

5 groups of 3

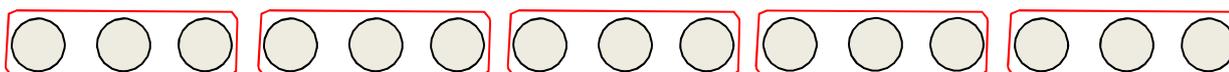
3 multiplied by 5

$$3 \times 5$$

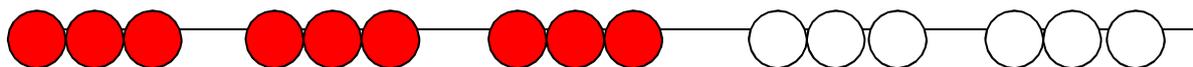
3 multiplied by 5 can be shown as five groups of three with counters, either grouped in a random pattern, as below:



or in a more ordered pattern, with the groups of three indicated by the border outline:

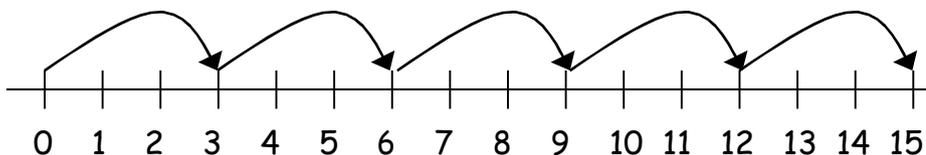


**100 Bead strings** or **bead bars** can be used to model repeated addition:  
**Number sentences** are always recorded.



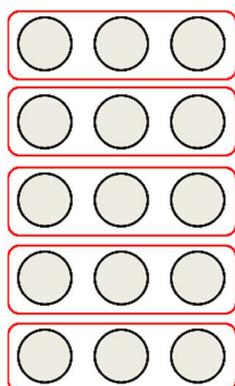
$$3 + 3 + 3 + 3 + 3 = 15$$

+ 3                    + 3                    + 3                    + 3                    + 3



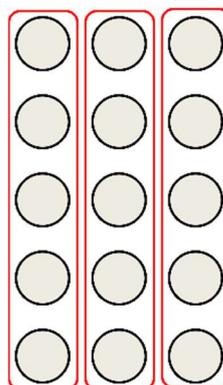
Children should then develop this knowledge to show how multiplication calculations can be represented by an array, (this knowledge will support with the development of the grid method in the future). Again, children should be encouraged to use **practical apparatus** and jottings to support their understanding, e.g.

3 x 5 can be represented as an array in two forms (as it has commutativity):



$$3 + 3 + 3 + 3 + 3 = 15$$

$$3 \times 5 = 15$$



$$5 + 5 + 5 = 15$$

$$5 \times 3 = 15$$

Once children understand the commutative order of multiplication the order is irrelevant.

Partial tables/key facts box should be introduced every time children have learnt, understood and can recall multiplication facts. This method will also support doubling, halving, adding and subtracting.

1x	3
2x	6
4x	12
5x	15
10x	30
20x	60

Groups of 3  
 Partial tables/key  
 facts box

1x	4
2x	8
4x	16
5x	20
10x	40
20x	80

Groups of 4  
 Partial tables/key  
 facts box

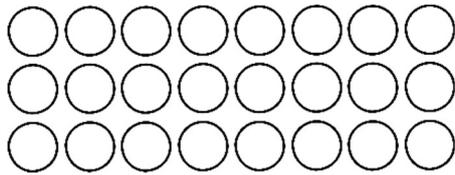
1x	8
2x	16
4x	32
5x	40
10x	80
20x	160

Groups of 8  
 Partial tables/key  
 facts box

To progress children in Y3 they will initially, children will continue to use arrays where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.

$$3 \times 8$$

They may show this using practical equipment:



$$3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 3 \times 8 = 24$$

$$8 + 8 + 8 = 8 \times 3 = 24$$

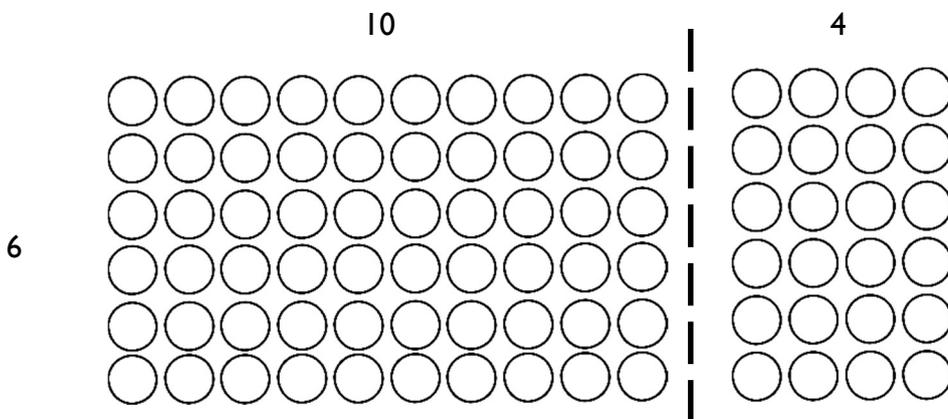
or by jottings using squared paper:

	x	x	x	x	x	x	x	x	
	x	x	x	x	x	x	x	x	
	x	x	x	x	x	x	x	x	

$$3 + 3 + 3 + 3 + 3 + 3 + 3 + 3 = 3 \times 8 = 24$$

$$8 + 8 + 8 = 8 \times 3 = 24$$

As they progress to multiplying a two-digit number by a single digit number, children should use their knowledge of partitioning two digit numbers into tens and units/ones to help them. For example, when calculating  $14 \times 6$ , children should set out the array, then partition the array so that one array has ten columns and the other four.

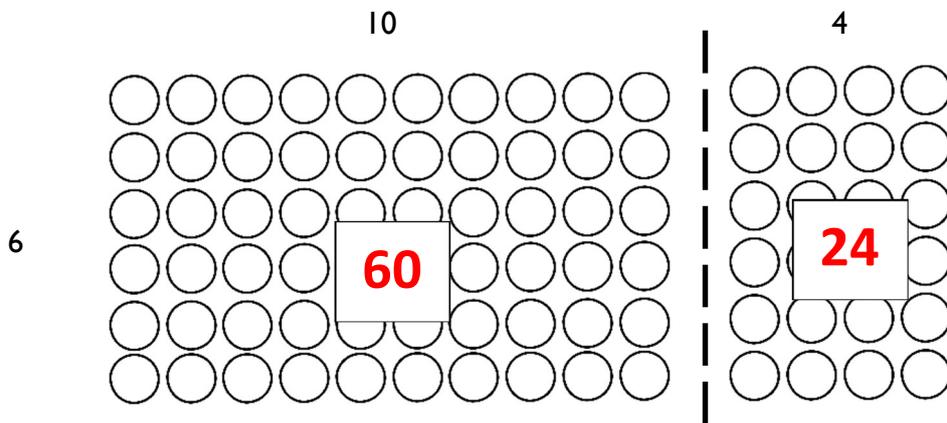


Partitioning in this way, allows children to identify that the first array shows  $10 \times 6$  and the second array shows  $4 \times 6$ . These can then be added to calculate the answer:

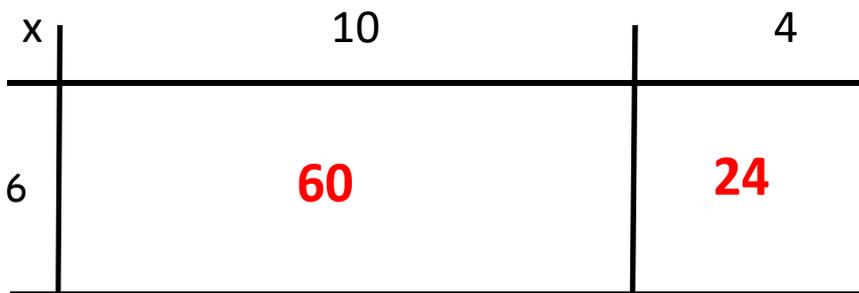
$$\begin{aligned} &(10 \times 6) + (4 \times 6) \\ &= 60 + 24 \\ &= 84 \end{aligned}$$

NB There is no requirement for children to record in this way, but it could be used as a jotting to support development if needed.

This method is the precursor step to the grid method. Using a two-digit by single digit array, they can partition as above, identifying the number of rows and the number of columns each side of the partition line.



By placing a box around the array, as in the example below, and by removing the array, the grid method can be seen.



It is really important that children are confident with representing multiplication statements as arrays and understand the rows and columns structure before they develop the written method of recording. From this, children can use the grid method to calculate two-digit by one-digit multiplication calculations, initially with two digit numbers less than 20. Children should be encouraged to set out their addition in a column at the side to ensure the place value is maintained. When children are working with numbers where they can confidently and correctly calculate the addition mentally, they may do so.

13 x 8

x	10	3
8	80	24

$$\begin{array}{r}
 80 \\
 + 24 \\
 \hline
 104
 \end{array}$$

When children are ready, they can then progress to using this method with other two-digit numbers.

37 x 6

x	30	7
6	180	42

$$\begin{array}{r}
 180 \\
 + 42 \\
 \hline
 222
 \end{array}$$

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

## End of Y4 Objectives

Recall multiplication and division facts for multiplication tables up to  $12 \times 12$ .

Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers.

Recognise and use factor pairs and commutativity in mental calculations.

Multiply two-digit and three-digit numbers by a one-digit number using formal written layout.

Solve problems involving multiplying and adding, including using the distributive law and harder multiplication problems such as which  $n$  objects are connected to  $m$  objects.

*PLEASE SEE GUIDANCE FROM Y2 AND Y3 AND ENSURE CHILDREN ARE CONFIDENT AND SECURE IN MULTIPLICATION OBJECTIVES AND UNDERSTANDING.*

*The process of **concrete**, **pictorial** and **symbolic** should be explored every children are introduced to a multiplication family e.g. repeated addition, arrays, commutativity and written number sentences for the remaining multiplication tables introduced in Y4 e.g. 6, 7, 9, 11 and 12.*

*Partial tables/key facts box should also be introduced for each multiplication family to support children in manipulating numbers and towards a more efficient mental and written method. This method will also support doubling, halving, adding and subtracting.*

*\*Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4.*

*It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.*

Children will move to Y4 using whichever method they were using as they transitioned from Y3. They will further develop their knowledge of the grid method to multiply any two-digit by any single-digit number, e.g.

79 x 8

x	70	9
8	560	72

$$\begin{array}{r} 560 \\ + 72 \\ \hline 632 \end{array}$$

To support the grid method, children should develop their understanding of place value and facts that are linked to their knowledge of tables. For example, in the calculation above, children should use their knowledge that  $7 \times 8 = 56$  to know that  $70 \times 8 = 560$ .

By the end of the year, they will extend their use of the grid method to be able to multiply three-digit numbers by a single digit number, e.g.  $346 \times 8$

x	300	40	6
8	2400	320	48

$$\begin{array}{r} 2400 \\ + 320 \\ + 48 \\ \hline 2768 \end{array}$$

When children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they may do so.

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

**End of Y5 Objectives**

**Multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers.**

**Multiply and divide numbers mentally drawing upon known facts.**

**Multiply & divide whole numbers & those involving decimals by 10, 100 & 1000.**

*PLEASE SEE GUIDANCE FROM Y2, Y3 and Y4 AND ENSURE CHILDREN ARE CONFIDENT AND SECURE IN MULTIPLICATION OBJECTIVES AND UNDERSTANDING.*

*Partial tables/key facts box should also be 'kept bubbling' for each multiplication family to support children in manipulating numbers and towards a more efficient mental and written method. This method will also support doubling, halving, adding and subtracting.*

*\*Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4.*

*It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.*

Children should continue to use the grid method and extend it to multiplying numbers with up to four digits by a single digit number, e.g. 4346 x 8

x	4 000	300	40	6
8	32 000	2400	320	48

$$\begin{array}{r}
 32000 \\
 + 2400 \\
 + 320 \\
 + 48 \\
 \hline
 34768
 \end{array}$$

and numbers with up to four digits by a two-digit number, e.g. 2693 x 24

x	2000	600	90	3
20	40000	12000	1800	60
4	8000	2400	360	12

$$\begin{array}{r}
 40000 \\
 + 8000 \\
 + 12000 \\
 + 2400 \\
 + 1800 \\
 + 360 \\
 + 60 \\
 + 12 \\
 \hline
 64632
 \end{array}$$

When children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they may do so.

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

### End of Y6 Objectives

**Multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication.**

*PLEASE SEE GUIDANCE FROM Y2, Y3, Y4 and Y5 AND ENSURE CHILDREN ARE CONFIDENT AND SECURE IN MULTIPLICATION OBJECTIVES AND UNDERSTANDING.*

*Partial tables/key facts box should also be 'kept bubbling' for each multiplication family to support children in manipulating numbers and towards a more efficient mental and written method. This method will also support doubling, halving, adding and subtracting.*

*\*Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4.*

*It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.*

By the end of Y6, children should be able to use the grid method to multiply any number by a two-digit number. They should also develop the method to be able to multiply decimal numbers with up to two decimal places, e.g.

$$4.92 \times 3$$

x	4	0.9	0.02
3	12	2.7	0.06

$$\begin{array}{r} 12 \\ + 2.7 \\ + 0.06 \\ \hline 14.76 \end{array}$$

When children are working with numbers where they can confidently and correctly calculate the addition (or parts of the addition) mentally, they may do so.

Children should also be using this method to solve problems and multiply numbers, including those with decimals, in the context of money or measures, e.g. to calculate the cost of 7 items at £8.63 each, or the total length of six pieces of ribbon of 2.28m each.

## PROGRESSION TOWARDS A WRITTEN METHOD FOR DIVISION

In developing a written method for division, it is important that children understand the concept of division, in that it is:

- repeated subtraction (inverse of repeated addition)
- sharing into equal amounts

Also, to understand and work with certain principles:

- the inverse of multiplication (linked to arrays)
- not commutative i.e.  $15 \div 3$  is not the same as  $3 \div 15$
- not associative i.e.  $30 \div (5 \div 2)$  is not the same as  $(30 \div 5) \div 2$
- = sign means same value but 'looks different'

**Always ensure that children are provided with an opportunity to access**

- ✓ Concrete
- ✓ Pictorial representation
- ✓ Symbolic

**DO – TALK – RECORD**

### **Early Learning Goal:**

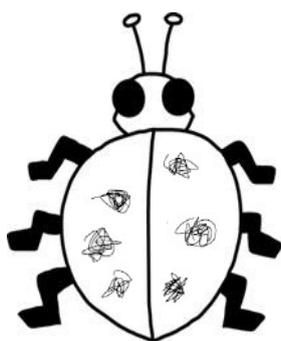
**Children solve problems including doubling, halving and sharing.**

*Children are encouraged to develop a **mental picture** of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of **equipment**, including small world play, role play, counters, cubes etc.*

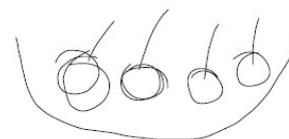
Children may also investigate sharing items or putting items into groups using items such as egg boxes, ice cube trays and baking tins which are arrays.



They may develop ways of recording calculations using pictures, etc.



A child's jotting showing halving six spots between two sides of a ladybird.



A child's jotting showing how they shared the apples at snack time between two groups.



### End of Y1 Objectives:

Solve simple one-step problems involving multiplication and division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher.

In Y1, children will continue to solve division problems using **practical equipment** and jottings of pictorial recordings and recording number sentences if appropriate.

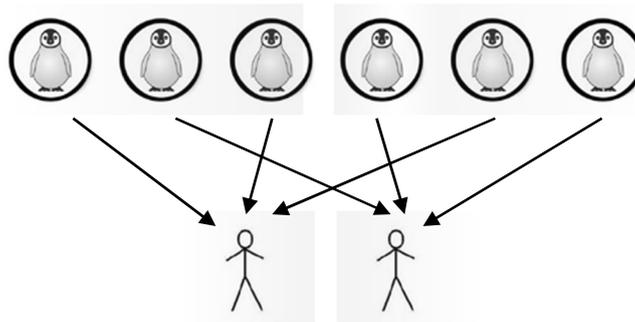
There are two main models of division at this early stage – sharing and grouping.

Through sharing and grouping of small quantities, pupils should begin to understand division and its relationship with multiplication.

### Sharing

Strategy – ‘one for you, one for me’ method until all of the objects have been given out.

They should use the equipment to share objects answering questions such as ‘If six football stickers are shared between two people, how many do they each get?’ Children should find the answer by counting how many each person has got.



### Grouping

Strategy – repeated subtraction of groups of objects from the original amount.

They should use the equipment to group objects answering questions such as ‘There are six football stickers, how many people can have 2 stickers each?’ Children should find the answer by counting how many groups of two there are.



When teaching it is important to understand the division number sentence as this dictates the model of division used to calculate.

$$\text{dividend} \div \text{divisor} = \text{quotient}$$

If the dividend and the quotient are the same unit it is SHARING e.g. the chickens have laid 10 eggs. 5 people buy the eggs. How many eggs do they have each?

If the dividend and quotient are different units it is GROUPING e.g. the chickens have laid 10 eggs. Eggs must be packed into boxes of 5. How many boxes are needed?

## Remainders

Children should be introduced to the concept of simple remainders in their calculations at this practical stage, being able to identify that the groups are not equal and should refer to the remainder as '... left over'.

### End of Y2 Objectives:

Recall and use multiplication and division facts for the 2, 5 and 10 multiplication tables, including recognising odd and even numbers.

Calculate mathematical statements for multiplication and division within the multiplication tables and write them using the multiplication ( $\times$ ), division ( $\div$ ) and equals ( $=$ ) signs.

Show that multiplication of two numbers can be done in any order (commutative) and division of one number by another cannot.

Solve one-step problems involving multiplication and division, using materials, arrays, repeated addition, mental methods, and multiplication and division facts, including problems in contexts.

The following process of concrete, pictorial and symbolic should be explored every children are introduced to a division family.

Children should understand and be able to calculate division as repeated subtraction, supported by:

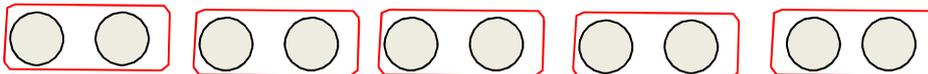
**Concrete** – counters, cubes and 100 bead strings

**Pictorial representation** – number lines and empty number line

**Symbolic** – written number sentence

Children will utilise practical equipment to represent division calculations as grouping (repeated subtraction) and use jottings to support their calculation, e.g.

$$10 \div 2 = 5$$

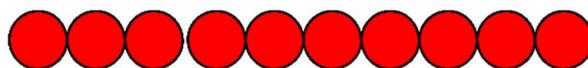


Children need to understand that this calculation reads as 'How many groups of 2 are there in 10?'

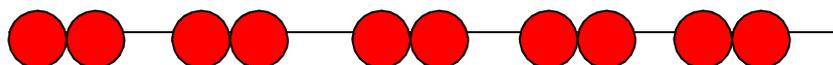
**100 Bead strings** or **bead bars** can be used to model repeated subtraction:

How many groups of 2 are there in 10?

Step 1

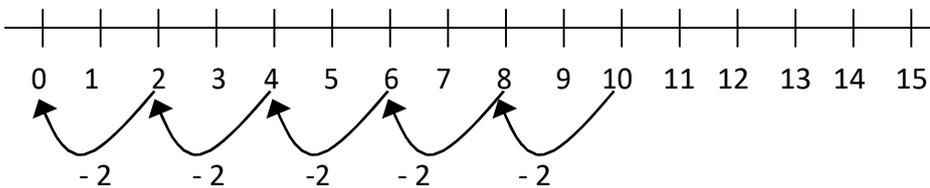


Step 2

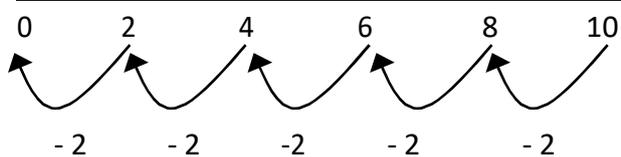


**Number lines** can be used to model and record repeated subtraction:

**Number sentences** are always recorded e.g.  $10 \div 2 = 5$

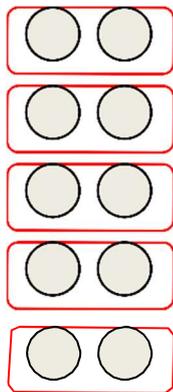


**Number lines** will be progressed into **empty number lines** to model and record repeated subtraction:  
**Number sentences** are always recorded e.g.  $10 \div 2 = 5$

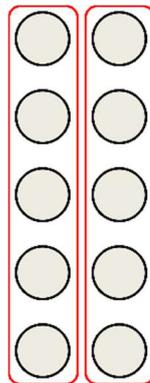


Children should then develop this knowledge to show how division calculations can be represented by an array. Again, children should be encouraged to use **practical apparatus** and jottings to support their understanding, e.g.

$10 \div 2 = 5$  can be represented as an array in two forms:



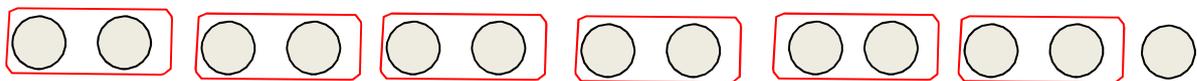
10 divided into groups of 2 = 5  
 $10 \div 2 = 5$



10 divided into groups of 5 = 2  
 $10 \div 5 = 2$

### Remainders

They should also continue to develop their knowledge of division with remainders, e.g.  
 $13 \div 2 = 5$  remainder 1



Children need to be able to make decisions about what to do with remainders after division and round up or down accordingly. In the calculation  $13 \div 2$ , the answer is 6 remainder 1, but whether the answer should be rounded up to 7 or rounded down to 6 depends on the context, as in the examples below:

I have £13. Books are £2 each. How many can I buy?

Answer: 6 (the remaining £1 is not enough to buy another book)

Apples are packed into boxes of 2. There are 13 apples. How many boxes are needed?

Answer: 7 (the remaining 1 apple still needs to be placed into a box)

### End of Y3 Objectives

Recall and use multiplication and division facts for the 3, 4 and 8 multiplication tables.

Write and calculate mathematical statements for multiplication and division using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods.

Solve problems, including missing number problems, involving multiplication and division, including integer scaling problems and correspondence problems in which  $n$  objects are connected to  $m$  objects.

*Please see guidance from Y2 and ensure children are confident and secure in division objectives and understanding.*

*\*Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4. It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.*

The following process of concrete, pictorial and symbolic should be explored every children are introduced to a division family e.g. dividing by 3, 4 and 8.

Children should understand and be able to calculate multiplication as repeated subtraction, supported by:

**Concrete** – counters, cubes and 100 bead strings

**Pictorial representation** – number lines and empty number line

**Symbolic** – written number sentence

$$12 \div 3 =$$



Children need to understand that this calculation reads as 'How many groups of 3 are there in 12?'

They should also continue to develop their knowledge of division with remainders, e.g.

$$13 \div 4 =$$



### Progressing in Y3

In preparation for developing the 'chunking' method of division, children should first use the repeated subtraction on a vertical number line alongside the continued use of practical equipment. There are two stages to this:

**Stage 1** – repeatedly subtracting individual groups of the divisor

**Stage 2** – subtracting multiples of the *divisor* (initially 10 groups and individual groups, then 10 groups and other multiples in line with table's knowledge)

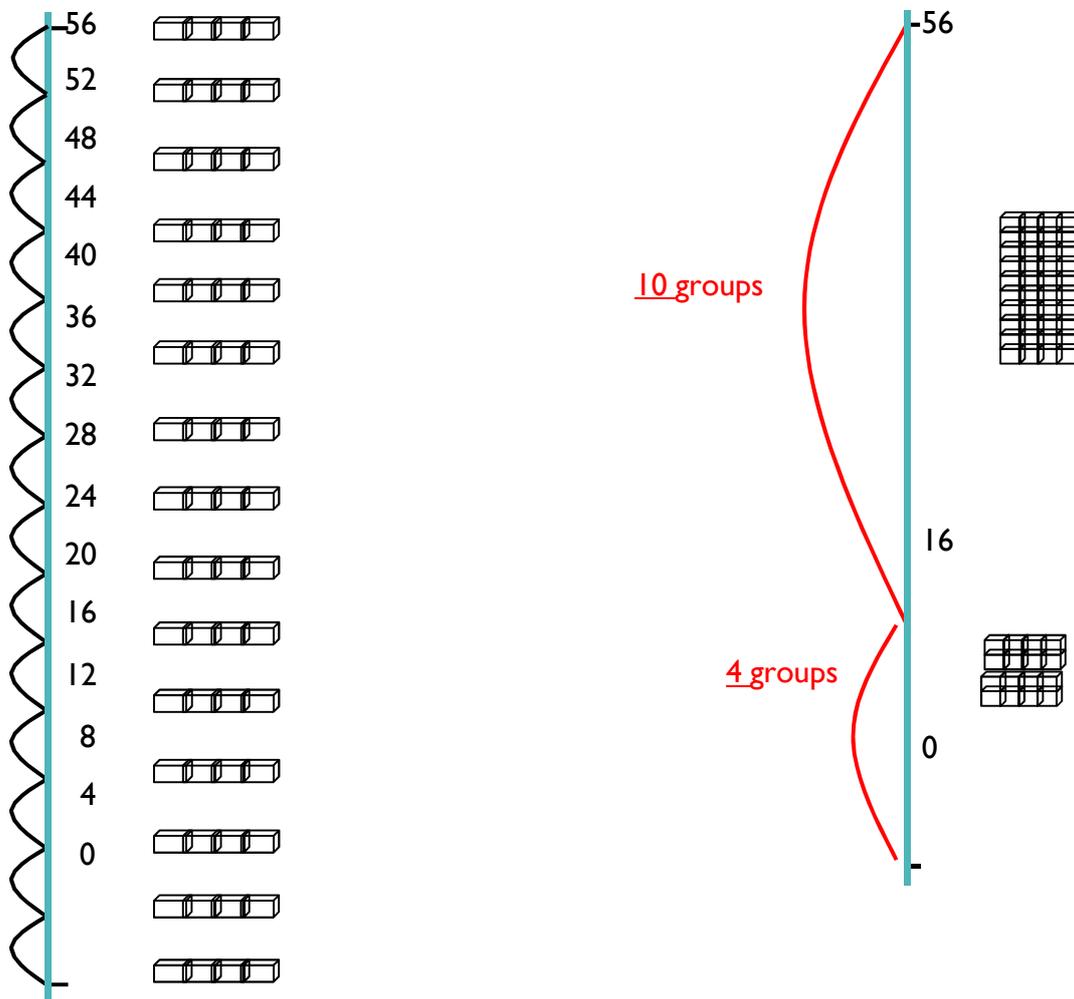
After each group has been subtracted, children should consider how many are left to enable them to identify the amount remaining on the number line.

**Stage 1**

$56 \div 4 = 14$  (groups of 4)

**Stage 2**

$56 \div 4 = 10$  (groups of 4) +  $4$  (groups of 4)  
 $= 14$  (groups of 4)



Partial tables/key facts box should be used to support children in recall of multiplication facts to support efficient mental and written calculations in division. It will help them to identify the largest group they can subtract in one chunk.

1x	3
2x	6
4x	12
5x	15
10x	30
20x	60

Groups of 3  
 Partial tables/key facts box

1x	4
2x	8
4x	16
5x	20
10x	40
20x	80

Groups of 4  
 Partial tables/key facts box

1x	8
2x	16
4x	32
5x	40
10x	80
20x	160

Groups of 8  
 Partial tables/key facts box

**Reminders**

Initially, children will continue to use division by grouping (including those with remainders), where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.  $43 \div 8 = 5$  remainder 3.



Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

### End of Y4 Objectives

**Recall multiplication and division facts for multiplication tables up to  $12 \times 12$ .**

**Use place value, known and derived facts to multiply and divide mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers.**

**Divide numbers up to 3 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.**

**Solve problems involving multiplying and adding, including using the distributive law and harder multiplication problems such as which  $n$  objects are connected to  $m$  objects.**

*PLEASE SEE GUIDANCE FROM Y2 AND Y3 AND ENSURE CHILDREN ARE CONFIDENT AND SECURE IN DIVISION OBJECTIVES AND UNDERSTANDING.*

*The process of **concrete**, **pictorial** and **symbolic** should be explored every children are introduced to a division family e.g. repeated subtraction, arrays and written number sentences for the remaining division families introduced in Y4 e.g. 6, 7, 9, 11 and 12.*

*Partial tables/key facts box should also be introduced for each multiplication family to support children in manipulating numbers and towards a more efficient division mental and written method. This method will also support doubling, halving, adding and subtracting.*

*\*Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4.*

*It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.*

Children will continue to develop their use of grouping (repeated subtraction) to be able to subtract multiples of the divisor, moving on to the use of the 'chunking' method. Partial tables/key facts box should be used to support children in recall of multiplication facts to support efficient mental and written calculations in division. It will help them to identify the largest group they can subtract in one chunk.

1x	6
2x	12
4x	24
5x	30
10x	60
20x	120

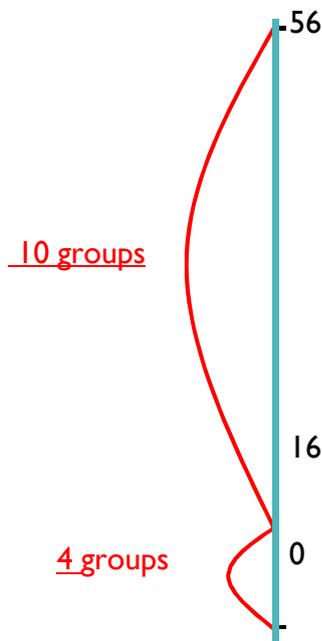
Groups of 6  
Partial tables/key  
facts box

1x	7
2x	14
4x	28
5x	35
10x	70
20x	140

Groups of 7  
Partial tables/key  
facts box

1x	9
2x	18
4x	36
5x	45
10x	90
20x	180

Groups of 9  
Partial tables/key  
facts box



$$\begin{array}{r}
 14 \\
 4 \overline{) 56} \\
 \underline{- 40} \\
 16 \\
 \underline{- 16} \\
 0
 \end{array}$$

Answer: 14

Children should write their answer above the calculation to make it easy for them and the teacher to distinguish.

Remember - The number line method used in Y3 can be linked to the chunking method to enable children to make links in their understanding.

When developing their understanding of 'chunking', children should utilise a 'key facts' box, as shown below. This will help them in identifying the largest group they can subtract in one chunk rather than taking chunks of 10 groups each time which is not efficient. Any remainders should be shown as integers, e.g.  $73 \div 3 = 24$  remainder 1.

$$\begin{array}{r}
 24r1 \\
 3 \overline{) 73} \\
 \underline{- 30} \\
 43 \\
 \underline{- 30} \\
 13 \\
 \underline{- 6} \\
 7 \\
 \underline{- 6} \\
 1
 \end{array}$$

1x	3
2x	6
4x	12
5x	15
10x	30
20x	60

Groups of 3  
Partial tables/key facts box

$$\begin{array}{r}
 24r1 \\
 3 \overline{) 73} \\
 \underline{- 60} \\
 13 \\
 \underline{- 12} \\
 1
 \end{array}$$

Model and discuss with children how their calculations can become more compact and efficient.

By the end of Y4, children should be able to use the chunking method to divide a three digit number by a single digit number e.g.  $196 \div 6 = 32$  remainder 4.

$$\begin{array}{r}
 32r4 \\
 6 \overline{) 196} \\
 \underline{- 120} \\
 76 \\
 \underline{- 60} \\
 16 \\
 \underline{- 12} \\
 4
 \end{array}$$

1x	6
2x	12
4x	24
5x	30
10x	60
20x	120

Groups of 6  
Partial tables/key facts box

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

## End of Y5 Objectives

Divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context.

Multiply and divide numbers mentally drawing upon known facts.

Multiply & divide whole numbers & those involving decimals by 10, 100 & 1000.

PLEASE SEE GUIDANCE FROM Y2, Y3 and Y4 AND ENSURE CHILDREN ARE CONFIDENT AND SECURE IN DIVISION OBJECTIVES AND UNDERSTANDING.

Partial tables/key facts box should also be 'kept bubbling' for each multiplication family to support children in manipulating numbers and towards a more efficient division mental and written method. This method will also support doubling, halving, adding and subtracting.

\*Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4.

It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

Children may continue to use the key facts box for as long as they find it useful. Using their knowledge of linked multiplication facts, children should be encouraged to use higher multiples of the divisor. Any remainders should be shown as integers, e.g.  $523 \div 8 = 65$  remainder 3.

$\begin{array}{r} 65r3 \\ 8 \overline{) 523} \\ - 320 \\ \hline 203 \\ - 160 \\ \hline 43 \\ - 40 \\ \hline 3 \end{array}$	<table border="1"><tbody><tr><td>1x</td><td>8</td></tr><tr><td>2x</td><td>16</td></tr><tr><td>4x</td><td>32</td></tr><tr><td>5x</td><td>40</td></tr><tr><td>10x</td><td>80</td></tr><tr><td>20x</td><td>160</td></tr><tr><td>40x</td><td>320</td></tr></tbody></table>	1x	8	2x	16	4x	32	5x	40	10x	80	20x	160	40x	320	Groups of 8 Partial tables/key facts box
1x	8															
2x	16															
4x	32															
5x	40															
10x	80															
20x	160															
40x	320															

By the end of year 5, children should be able to use the chunking method to divide a four digit number by a single digit number. If children still need to use the key facts box, it can be extended to include 100x e.g.  $2458 \div 7 = 351$  remainder 1.

$\begin{array}{r} 351r1 \\ 7 \overline{) 2458} \\ - 2100 \\ \hline 358 \\ - 350 \\ \hline 8 \\ - 7 \\ \hline 1 \end{array}$	<table border="1"><tbody><tr><td>1x</td><td>7</td></tr><tr><td>2x</td><td>14</td></tr><tr><td>4x</td><td>28</td></tr><tr><td>5x</td><td>35</td></tr><tr><td>10x</td><td>70</td></tr><tr><td>20x</td><td>140</td></tr><tr><td>40x</td><td>280</td></tr></tbody></table>	1x	7	2x	14	4x	28	5x	35	10x	70	20x	140	40x	280	<table border="1"><tbody><tr><td>100x</td><td>700</td></tr><tr><td>200x</td><td>1400</td></tr><tr><td>400x</td><td>2800</td></tr><tr><td>500x</td><td>35000</td></tr></tbody></table>	100x	700	200x	1400	400x	2800	500x	35000	Groups of 7 Partial tables/key facts box
1x	7																								
2x	14																								
4x	28																								
5x	35																								
10x	70																								
20x	140																								
40x	280																								
100x	700																								
200x	1400																								
400x	2800																								
500x	35000																								

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

### End of Y6 Objectives

**Divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context.**

**Use written division methods in cases where the answer has up to two decimal places.**

*PLEASE SEE GUIDANCE FROM Y2, Y3, Y4 and Y5 AND ENSURE CHILDREN ARE CONFIDENT AND SECURE IN DIVISION OBJECTIVES AND UNDERSTANDING.*

*Partial tables/key facts box should also be 'kept bubbling' for each multiplication family to support children in manipulating numbers and towards a more efficient division mental and written method. This method will also support doubling, halving, adding and subtracting.*

*\*Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4.*

*It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.*

To develop the chunking method further, it should be extended to include dividing a four-digit number by a two-digit number, e.g.  $6367 \div 28 = 227$  remainder 11.

$$\begin{array}{r} 227r11 \\ 28 \overline{)6367} \\ \underline{-5600} \quad 200x \\ 767 \\ \underline{-560} \quad 20x \\ 207 \\ \underline{-140} \quad 5x \\ 67 \\ \underline{-56} \quad 2x \\ 11 \end{array}$$

1x	28
2x	56
4x	112
5x	140
10x	280
20x	540

100x	2800
200x	5600
400x	11200
500x	14000

Groups of 28  
Partial tables/key  
facts box

Children should be able to solve real life problems including those with money and measures. They need to be able to make decisions about what to do with remainders after division and round up or down accordingly.

In addition, children should also be able to use the chunking method and solve calculations interpreting the remainder as a decimal up to two decimal places, e.g.  $362 \div 17$

