Rationale

In the report entitled 'Good Practice in Primary Mathematics: Evidence from 20 successful schools' (2011: 7), Ofsted states that:

A feature of strong practice in maintained schools is their clear, coherent calculation policies and guidance, which are tailored to the particular school's context. They ensure consistent approaches and use of visual images and models that secure progression in pupils' skills and knowledge lesson by lesson and year by year.

Our approach to the teaching of calculation is guided by the following principles while embedding White Rose:

Practical, representational activities are essential in the development of the mathematical concepts needed for calculation, in line with Bruner's concrete-pictorial-abstract theory.

Mental calculation is not at the exclusion of pictorial or written procedures and should be seen as complimentary to and not as separate from them.

In every pictorial or written procedure there is an element of mental processing.

Sharing pictorial or written methods with teaching and learning staff and their peers encourages children to think about the mental strategies that underpin them and to develop new ideas.

Representations and written recording help children to clarify their thinking and supports and extends the development of more fluent and sophisticated mental strategies.

Our ultimate aim is for children to be able to select an efficient method that is appropriate for a given task. Children will always be encouraged to look at a calculation or problem before deciding which the best method to use is. They will do this by asking themselves:

¨ Can I do this in my head?

¨ Can I do this mentally using equipment, drawings or jottings?

¨ Can I explain what I have done to somebody else?

¨ Can I use previous knowledge to support me?

Aims:

We feel that the 'Concrete – Pictorial – Abstract' approach is at the heart of effective Mathematics teaching. It provides children with a conceptual understanding of Mathematics and is a gradual and systematic approach which builds on existing knowledge. At St. Augustine's Academy, we promote the use of concrete apparatus to develop this conceptual understanding.

The purpose of this policy is to provide all stakeholders with guidance for the development of conceptual understanding in calculation methods. The policy attempts to ensure pupils receive a smooth progression in the development of calculation strategies, in which children's written calculation methods build effectively upon their mental methods. As a school, it is our aim for every child to develop efficient written methods for calculation for each of the four operations which they can use confidently, accurately and with an understanding of the mathematics involved. Children should not be made to go onto the next stage if they are not ready or confident with the preceding stage.

Development of language:

During their time at St Augustine's Academy, children will be encouraged to understand Mathematics as both a written and spoken language. The use of vocabulary is crucial to being successful in mathematics. Children need to be able to describe their calculation methods confidently and consistently using appropriate mathematical vocabulary. Teachers must carefully select the appropriate vocabulary for each lesson. This must be modelled and displayed accordingly for the children to access and use. Children must be encouraged to use the correct key vocabulary.

Using this document:

In the top left corner of each page an example of the written procedure at the end of each stage is given. A suggested concrete representation of this is shown below it along with further exemplifications of the CPA approach and stage appropriate problems. For developing Mastery within concepts, MATHSHUBS Enigma materials are used.

Addition - Progression in written methods - Stage 1

Key procedure:	Key skills:
Use a number track to count on	• **Number bonds to 10** • **Add numbers to 10 including zero** • **Recognise and use the addition and equals symbol (+, =)** • **Solve problems (including missing number problems)** • **Understand equality (=) and inequality (<,>)**

$4 + 2 = 6$

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |

Key Vocabulary:

$1 + 5 = 6$

addend (part) addend (part) sum (whole)
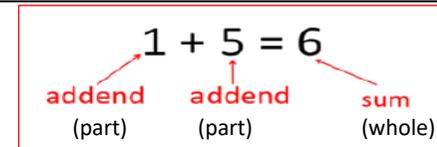
Addition, number bonds, add, more, plus, make, sum, total, altogether, inverse, double, near double, equals, is the same as, less than, greater than

Sentence stems: how many more to make…?, how many more is… than…?, how many more (greater is)…than…?

… is a part … is a part … is the whole

Concrete and pictorial progression to support Mastery.

Use **pictorial** representations - part whole model and ten frames
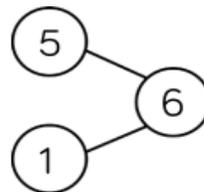
Missing number problems

Concrete

Start with expressions (no equals sign) progress to using =

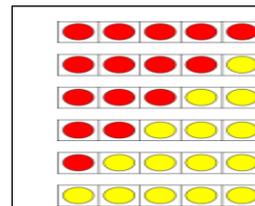Bead strings/base ten/ten frames to be used to support addition
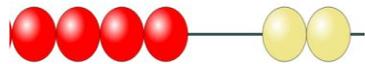
4   +   2 makes 6

4   +   2 = 6

5
6
1

5   +   1 =   6
part + part = whole

$5 = 5 + 0$
$5 = 4 + 1$
$\_ = \_ + \_$
$\_ = \_ + \_$
$\_ = \_ + \_$
$\_ = \_ + \_$

$4 + \_ = 10$ $5 + \_ = 10$
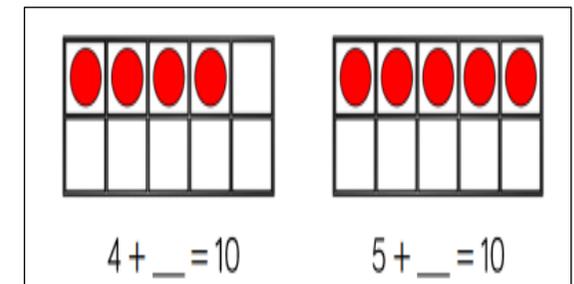
Begin to use the bar model

5	

Addition - Progression in written methods - Stage 2

Key procedure:	Key skills:

Key procedure:

Expanded method with exchanging of ones (2-digit numbers)

NB: Progress from partitioning both addends to more efficient partitioning of 1 addend.

Partitioning both addends	Partitioning one addend
26 + 37 ╱╲ ╱╲ 20 6 30 7	26 + 37 ╱╲ 30 7
$20 + 30 = 50$ $6 + 7 = 13$ $50 + 13 = 63$	$26 + 30 = 56$ $56 + 7 = 63$

Sentence stem:

Example: The sum of 6 ones and 7 ones is 13 ones.
This is exchanged into 1 ten and 3 ones.
So, 26 + 37 is equal to 6 tens and 3 ones, which is 63.

Key skills:
- **Number bonds to 20 and related facts 100**
- **Exchanging ones and tens (understanding place value)**
- **Add three one digit numbers**
- **Add two digit numbers to 1 and 2 digit numbers**
- **Understand addition is commutative (e.g. 1+2 = 2+1)**
- **Understand equality (=) and inequality (<,>)**
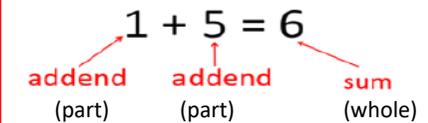
Key Vocabulary:

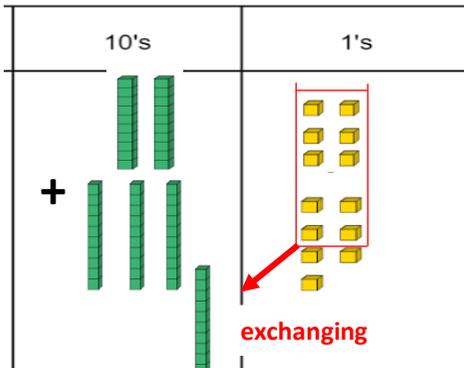$$1 + 5 = 6$$
addend (part)   addend (part)   sum (whole)

Addition, number bonds, add, more, plus, make, sum, total, altogether, inverse, double, near double, equals, is the same as, less than, greater than, exchange

Sentence stems: how many more to make…?, how many more is… than…?, how many more (greater is)…than…?
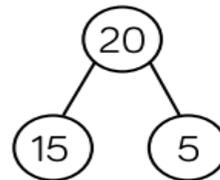
… is a part … is a part … is the whole

Concrete and pictorial progression to support Mastery.   Use **pictorial** representations - part whole model and ten frames   Solve number problems

Concrete

Base ten/bead strings/ten frames to be used
Example: 26 + 37 = 63 (exchanging)

10's	1's

exchanging

20
╱╲
15   5

$$15 + 5 = 20$$

part + part = whole

16   +   5
16   +   4 + 1   = 21

4 + 3 + 6

$4 + 6 = 10$

$10 + 3 = 13$

?		
52		17

50 2
+10 7
60 9

52 + 17 = 69

4
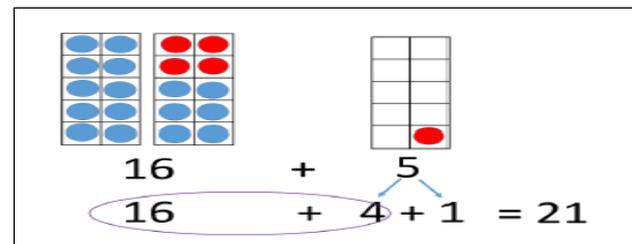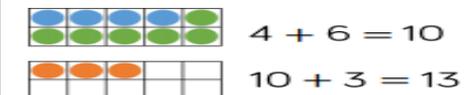
Addition - Progression in written methods - Stage 3

Key procedure:

Formal column addition exchanging tens and ones (up to three-digit numbers)

$$276 + 56 = 332$$

NB: Progress from exchanging ones only to exchanging ones into tens and tens into hundreds.

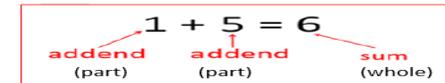Sentence stems - Column Method (exchanging and unitising)

Example: The sum of 6 ones and 6 ones is 12 ones this is exchanged into 1 ten and 2 ones.

```
    2  7  6
 +     5  6
 ---------
    3  3  2
 ---------
    1  1
```

Key skills:

- **Number bonds to 100**
- **Exchanging ones, tens and hundreds (understanding place value)**
- **Add three digit numbers to 1 and 2 and 3 digit numbers**
- **Estimate answers**
- **Use the inverse to check**
- **Understand equality (=) and inequality (<,>)**

Key Vocabulary:

$$1 + 5 = 6$$

addend (part)   addend (part)   sum (whole)

Addition, number bonds, add, more, plus, make, sum, total, altogether, inverse, double, near double, equals, is the same as, less than, greater than, exchange, estimate, rounding

Sentence stems: how many more to make…?, how many more is… than…?, how many more (greater is)…than…?

… is a part … is a part … is the whole
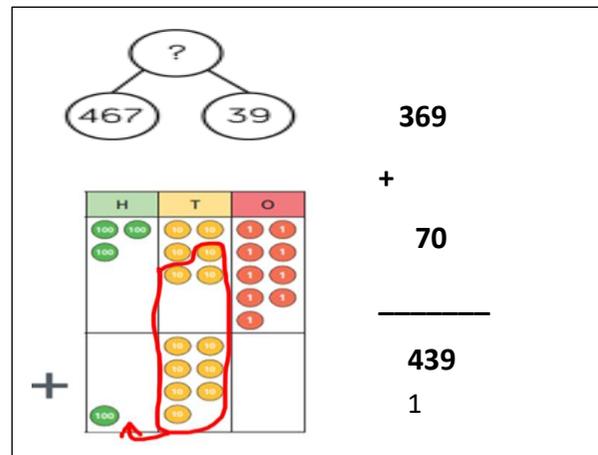
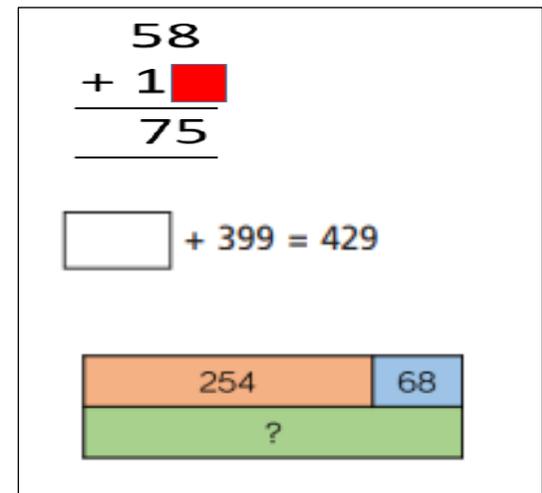### Concrete and pictorial progression to support Mastery.

Concrete

Base ten to be used -Example: 276 + 56 = 332 exchanging ones and tens.
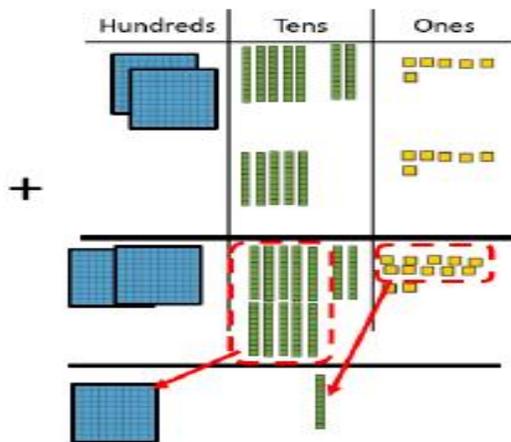
Use **pictorial** representations - part whole model/place value charts and counters

Solve number problems

$$369 + 70 = 439$$

```
    58
 +  1■
 ------
    75
```

$$\boxed{} + 399 = 429$$

254	68
?	

Addition - Progression in written methods - Stage 4

Key procedure:

Formal column addition exchanging hundreds, tens, ones (up to 4-digit numbers)

$$2534 + 577$$

NB: Progress from exchanging ones and tens to also exchange hundreds into thousands.
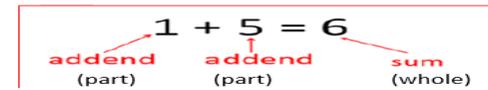
Sentence stems - Column Method (exchanging and unitising)

Example: The sum of 4 ones and 7 ones is 11 ones this is exchanged into 1 ten and 1 ones.

```
   2534
 +  577
 _____
   3111
   1 1 1
```

Key skills:

- **Exchanging ones, tens and hundreds and thousands.**
- **Add numbers up to 4 digits**
- **Estimate answers**
- **Use the inverse to check**
- **Understand equality (=) and inequality (<,>)**

Key Vocabulary:

$$1 + 5 = 6$$
addend (part)   addend (part)   sum (whole)

Addition, number bonds, add, more, plus, make, sum, total, altogether, inverse, double, near double, equals, is the same as, less than, greater than, exchange, estimate, rounding
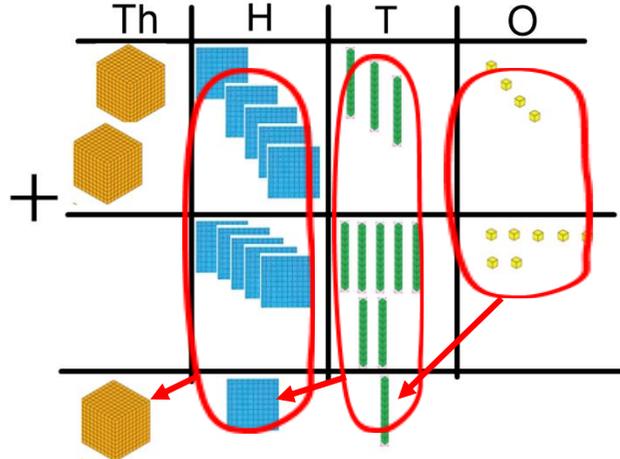
Sentence stems: how many more to make…?, how many more is… than…?, how many more (greater is)…than…?
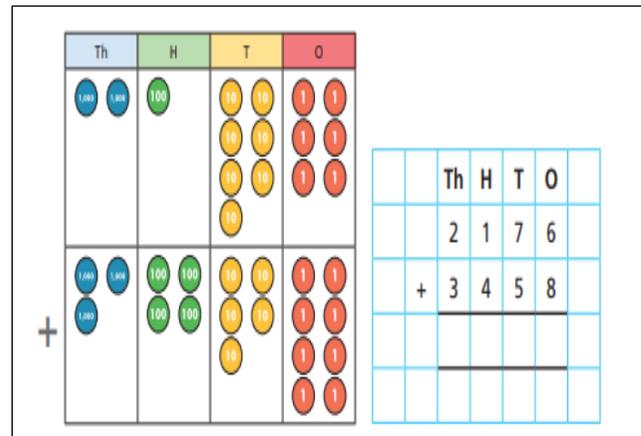
… is a part … is a part … is the whole

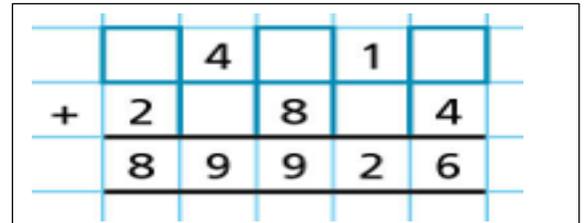**Concrete and pictorial progression to support Mastery.**

Concrete

Base ten to be used -Example: 2534 + 577 = 3111 **exchanging**..

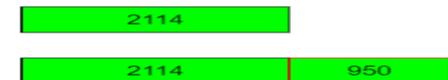Use **pictorial** representations - part whole model/place value charts and counters

Solve missing number problems

Solve multi-step problems (using the bar model)

There were 2114 visitors on Saturday. 950 fewer people visited on Saturday than Sunday. How many visited in total.

Addition - Progression in written methods – Stage 5

Key procedures:	Key skills:
Formal column addition exchanging thousands, hundreds, tens, ones (up to 5-digit numbers)	• **Exchanging ones, tens and hundreds and thousands.** • **Add numbers up to 5 digits** • **Add decimals with different number of digits** • **Estimate answers** • **Use the inverse to check**
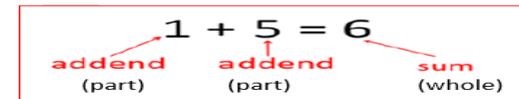
$$25342 + 5778$$

$$\begin{array}{r} 25342 \\ + \ 5778 \\ \hline 31120 \\ \hline \ 1\ 1\ 1\ 1 \end{array}$$

Key Vocabulary:

$$1 + 5 = 6$$
addend (part)   addend (part)   sum (whole)

Addition, number bonds, add, more, plus, make, sum, total, altogether, inverse, double, near double, equals, is the same as, less than, greater than, exchange, estimate, rounding, decimals

Sentence stems: how many more to make…?, how many more is… than…?, how many more (greater is)…than…?
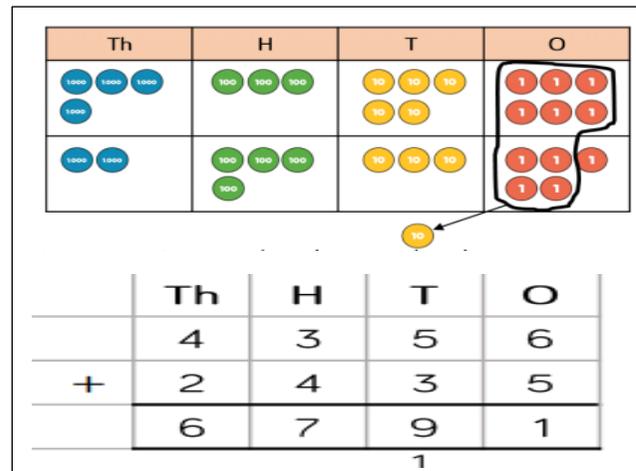
… is a part … is a part … is the whole

Concrete and pictorial progression to support Mastery.

Concrete

Place value chart and counters: 23245 + 12235 = 35480
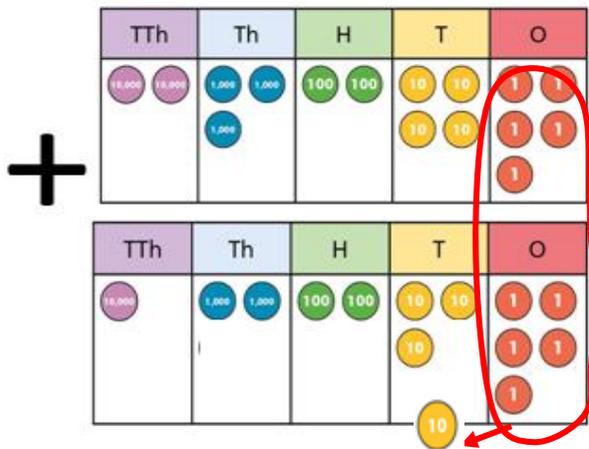
Use **pictorial** representations - part whole model/place value charts and counters

Th	H	T	O
4	3	5	6
+ 2	4	3	5
6	7	9	1
			1

Missing number problems

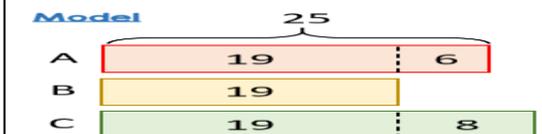|	Th	H	T	O
	6	?	?	8
+	?	?	8	?
	9	3	2	5

Multi-step problems using the bar model for comparison

There are 3 containers, A, B and C.
Each contain some water.
- B has 6 less litres than A
- A contains 25 litres of water.
- C contains 8 more litres of water than B
How many litres of water are in C?

Model   25

A	19	6
B	19	
C	19	8

Key procedure:

Formal column addition decimals with a different number of digits.

$$57.30$$
$$+\quad 6.08$$
$$\overline{63.38}$$
$$1$$

Key skill: Adding a place holder.
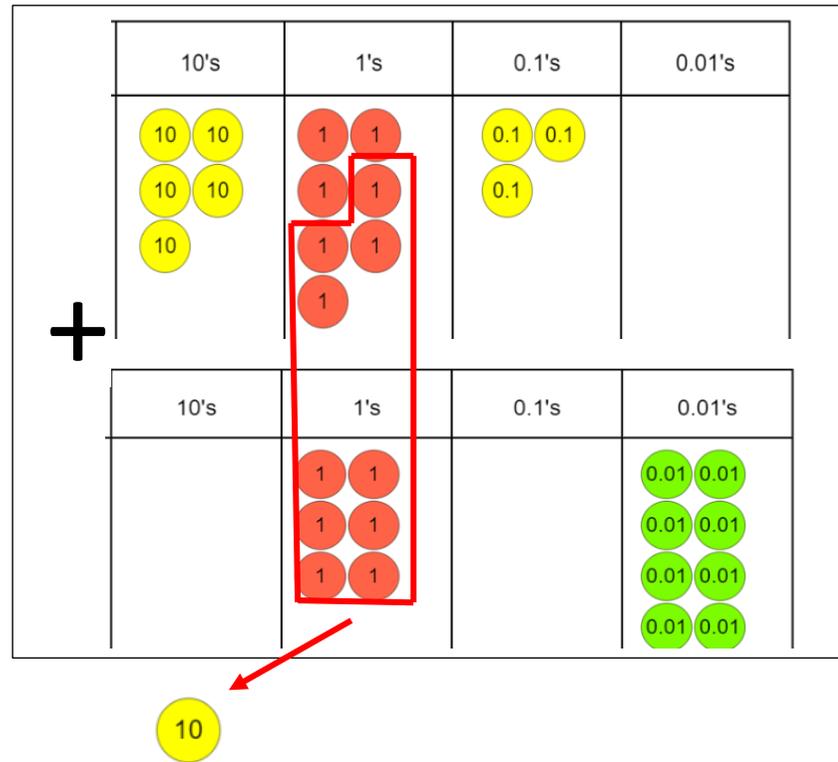
Subtraction: 3 key structures for subtraction.

Subtraction is a more complicated mathematical operation and as such has three separate structures. Many pupils' conceptual misunderstandings of subtraction originate in their lack of awareness of these structures.

Reduction structure

Reduction is the simplest structure and is the first to be introduced. Reduction concerns starting with a quantity and then this quantity getting smaller (for example, 'Katy starts with 5 sweets and eats 1, how many sweets does she have left?'). The First….then….now story model works well when teaching young children the reduction model: 'First Katy had 5 sweets, then she ate 1, now she has how many sweets?'.
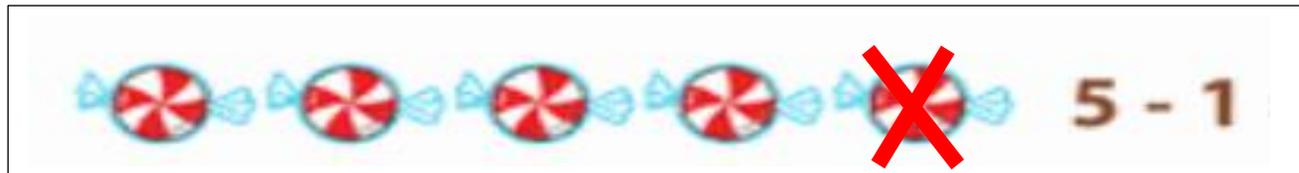

Reduction involving small quantities should be represented concretely and pictorially, with children physically moving objects away and crossing out drawings, before using a single bar model with the subtrahend (the number being taken away) crossed out.

Partitioning structure

Partitioning is different from reduction as nothing is being lost, sometimes confusion arises if children conclude that subtraction is only used when a quantity is being taken away. Partitioning questions demonstrate that this is not the case. When a subtraction question requires partitioning, the first number (minuend) is being split into two parts (the subtrahend and the difference). Crucially, nothing is being lost when partitioning occurs, yet subtraction is still necessary to calculate the answer.

For example, 'Katy has 7 sweets, some are marshmallows and the rest are lollipops. If 3 are lollipops, how many are marshmallows?'. No sweets are being eaten, thus 7 is not being reduced in magnitude. Rather, 7 is being split (partitioned) into two groups – lollipops and marshmallows.
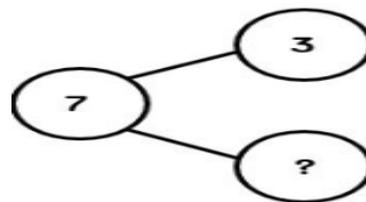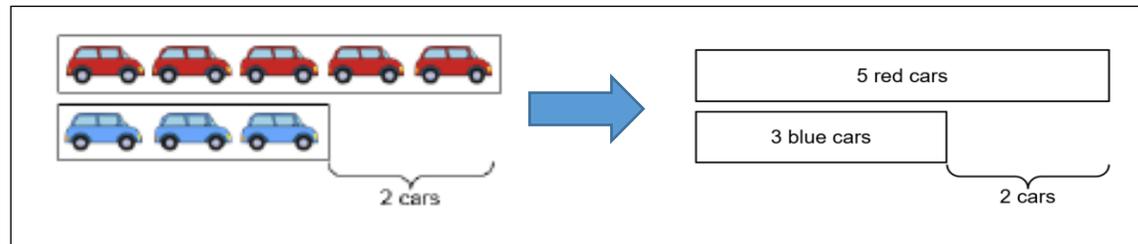
Subtraction

Difference structure

The (comparative) difference model of subtraction looks at the numerical difference between two groups. This is distinct from both reduction and partitioning because with difference problems the 'whole' (the whole number of cars, or the whole number of pupils in the class) is not relevant to the situation. This subtraction structure is therefore considerably removed from the reduction model (where the reduction of the whole is of central importance). A pictorial representation using a comparison bar model is helpful for pupils to use here, to avoid them relying on frequently misleading linguistic 'clues' in a difference question (the use of 'more' in the car example given above being such a clue).

Question: How many more red cars are there than blue cars?

As with all topics within Maths, the mathematical language we use (and which we should insist on children using) when teaching subtraction is of critical importance if pupils are to avoid all too common misconceptions; specifically, the mathematical symbol – means 'subtract', not 'take-away'.
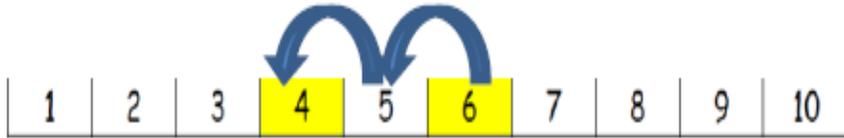
$$8 - 1 = 7$$

minuend   subtrahend   difference

NB- this would be the remaining quantity using the reduction structure.

Subtraction - Progression in written methods – Stage 1

Key procedure:

Use a number track to count back

$6 - 2 = 4$

1	2	3	4	5	6	7	8	9	10

Key skills:

- **Number bonds to 10 and related subtraction facts**
- **Subtract numbers within 10 including zero**
- **Recognise and use the subtraction and equals symbol (-, =)**
- **Solve problems (including missing number problems)**
- **Understand equality (=) and inequality (<,>)**

Key Vocabulary:

Subtract, take away, minus, decrease, fewer, difference, reduce, greater than, less than.

Sentence stems:

how many fewer is…than..?, how much less is..? half, halve, how many are left?, one less, two less…, how many fewer is… than…?, how much less is…? =, equals, is the same as, count on, count back, how many more is…than..? **is the whole … is a part ….is a part**

Concrete and pictorial progression to support Mastery.

Concrete
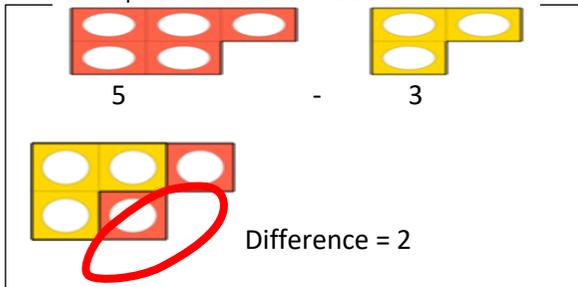
Start with expressions (no equals sign) progress to using =
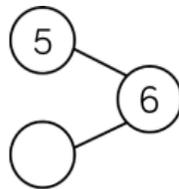Bead strings/base ten/ten frames/numicon to support subtraction.
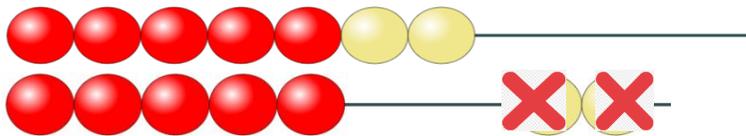
7   -   2 makes 5
7   -   2   =   5

Numicon to expose the **difference structure**

5   -   3

Difference = 2

Use **pictorial** representations – including part whole model and number

Partitioning structure

5 / 6

6   -   5   =   ?

whole - part = part

(minuend-subtrahend = difference)

First, then, now

There were 7 birds in a tree and 3 flew away.
Complete the sentences.

At first there were ___ birds. Then ___ flew away. Now there are ___ birds in the tree.

Use the number line to count back

0	1	2	3	4	5	6	7	8	9	10

$7 - 3 =$ _____
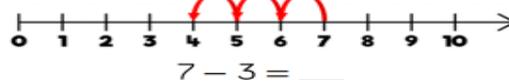
Finding the **difference**

How many more cakes does Whitney have than Teddy?

Whitney

Teddy

Whitney has ___ more cakes than Teddy.

Problem solving (including inequality).

What's the difference between 10 and 6?

The difference between 10 and 6 is ___

$10 - 6 =$ ___
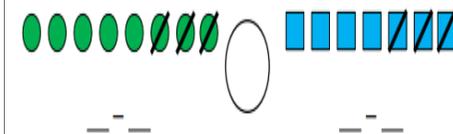
__ - __   __ - __

Subtraction - Progression in written methods - Stage 2

Key procedure:	Key skills:
Expanded method with exchanging of ones (2-digit numbers)	**Number bonds to 20 and related facts 100** **Subtract ones, tens and two digit numbers from two digit numbers.** **Partitioning 1 and 2 digit numbers** **Exchange tens and ones (understanding place value)** **Recognise and use the subtraction and equals symbol (-, =)** **Understand equality (=) and inequality (<,>)**

NB-Keep the first number (minuend) whole and partition the second number (subtrahend)

$$73 - 46 = \boxed{}$$

$$40 \qquad 6$$

$$73 - 40 = 33$$

$$33 - 6 = 27$$

Sentence stems -
Expanded Method (exchanging)

First I subtract the tens. Then I can see there are not enough ones so I need to exchange one ten into ten ones.

Key Vocabulary:
Subtract, take away, minus, decrease, fewer, difference, reduce, greater than, less than, inverse

Sentence stems:
how many fewer is…than..?, how much less is..? half, halve, how many are left?, one less, two less…, how many fewer is… than…?, how much less is…? =, equals, is the same as, count on, count back, how many more is…than..?
is the whole … is a part ….is a part

Concrete and pictorial progression to support Mastery. Use **pictorial** representations - part whole model and ten frames

Concrete
Base ten/bead strings/ten frames/numicon to be used
Example: 73-46 = 27 (exchanging)

Tens	Ones
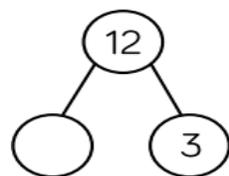
Partitioning structure

$$12 - 3 = ?$$

whole - part = part

(minuend-subtrahend = difference)

$$12 - 3 = 9$$

$$-3$$

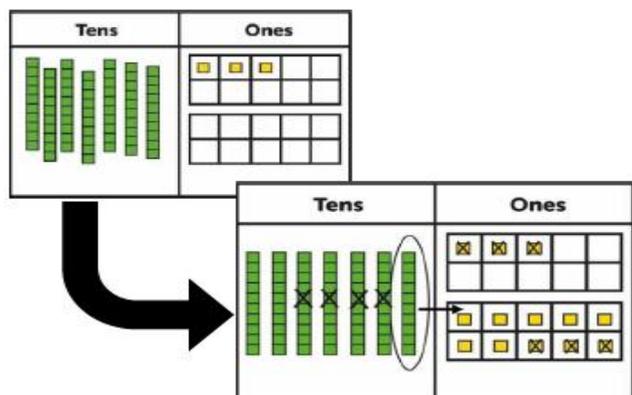0 1 2 3 4 5 6 7 8 9 10 11 12

Solve problems using related facts:

Understand inequality:
Find the greatest whole number that can complete each number sentence below.

$$45 - 17 > 14 + \underline{}$$

Find the difference:

Adults 26
Children 14 ?
} 40

Subtraction - Progression in written methods – Stage 3

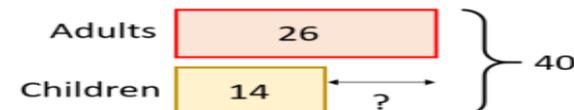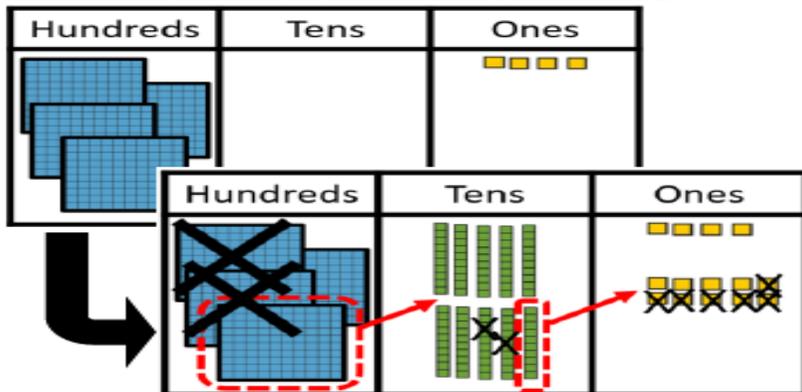|	
Key procedure: **Written column subtraction method with exchanging (3-digit numbers)** $$323 - 141 = 182$$ Sentence stems - Column Method (exchanging and unitising) I can see there are not enough … so I need to exchange one … into ten … 3 ones subtract 1 one is 2 ones 12 tens subtract 4 tens is 8 tens 2 hundreds subtract 1 hundred is one hundred.	**Key skills:** • **Subtraction up to 3 digit numbers.** • **Exchange hundreds, tens and ones (understanding place value)** • **Recognise and use the subtraction and equals symbol (-, =)** • **Estimate answers and use the inverse to check** • **Understand equality (=) and inequality (<,>)** **Key Vocabulary:** Subtract, take away, minus, decrease, fewer, difference, reduce, greater than, less than, inverse **Sentence stems:** how many fewer is…than..?, how much less is..? half, halve, how many are left?, one less, two less…, how many fewer is… than…?, how much less is…? =, equals, is the same as, count on, count back, how many more is…than..? I can see there are not enough … so I need to exchange one … into ten … **is the whole … is a part ….is a part**

Concrete and pictorial progression to support Mastery.   Use **pictorial** representations - part whole model and ten frames

Concrete
Base ten to be used
Example: 323-141 = 182 (exchanging)

Partitioning structure

132 − 37 = ?

whole - part = part

(minuend-subtrahend = difference)

Solve missing number problems:

Find the difference (in context):

Here are 3 buildings.
● A is 150 m tall
● B is 317 m taller than A
● C is 223 m shorter than B

How much taller is C than A?

Subtraction – Progression in written methods – Stage 3 (cont.)

Key procedure:

Written column subtraction method with exchanging <u>across multiple place value columns</u> (3-digit numbers)

$$404 - 226 = 178$$

$$\begin{array}{r} {}^3\cancel{4}\;{}^{\cancel{1}9}\cancel{0}\;{}^1 4 \\ -\;2\;\;2\;\;6 \\ \hline 1\;\;7\;\;8 \end{array}$$

Sentence stems - Column Method (exchanging and unitising)

I can see there are not enough … so I need to exchange one … into ten …

14 ones subtract 6 one is 8 ones

9 tens subtract 2 tens is 7 tens

3 hundreds subtract 2 hundreds is one hundred.

Key skills:
- **Subtraction up to 3 digit numbers.**
- **Exchange hundreds, tens and ones (understanding place value)**
- **Recognise and use the subtraction and equals symbol (-, =)**
- **Estimate answers and use the inverse to check**
- **Understand equality (=) and inequality (<,>)**

Key Vocabulary:
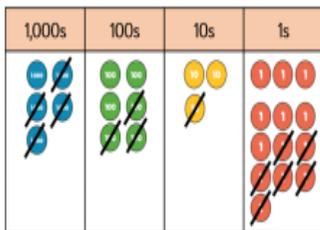Subtract, take away, minus, decrease, fewer, difference, reduce, greater than, less than, inverse

Sentence stems:
how many fewer is…than..?, how much less is..? half, halve, how many are left?, one less, two less…, how many fewer is… than…?, how much less is…? =, equals, is the same as, count on, count back, how many more is…than..?
I can see there are not enough … so I need to exchange one … into ten …
**is the whole … is a part ….is a part**

Concrete and pictorial progression to support Mastery.

14

Subtraction – Progression in written methods – Stage 4

Key procedure:
Written column subtraction method with exchanging across multiple place value columns (4-digit numbers)

5003 - 4316

$$
\begin{array}{r}
\overset{4}{\cancel{5}}\overset{9}{\cancel{0}}\overset{9}{\cancel{0}}\overset{1}{3} \\
- 4316 \\
\hline
687 \\
\hline
\end{array}
$$

Sentence stems - Column Method (exchanging and unitising)

I can see there are not enough … so I need to exchange one … into ten …

13 ones subtract 6 one is 7 ones

9 tens subtract 1 ten is 8 tens

9 hundreds subtract 3 hundreds is six hundreds.

4 thousands subtract 4 thousands is zero.

Key skills:
- **Subtraction up to 4 digit numbers.**
- **Exchange thousands, hundreds, tens and ones (understanding place value)**
- **Recognise and use the subtraction and equals symbol (-, =)**
- **Estimate answers and use the inverse to check**
- **Understand equality (=) and inequality (<,>)**

Key Vocabulary:
Subtract, take away, minus, decrease, fewer, difference, reduce, greater than, less than, inverse

Sentence stems:
how many fewer is…than..?, how much less is..? half, halve, how many are left?, one less, two less…, how many fewer is… than…?, how much less is…? =, equals, is the same as, count on, count back, how many more is…than..?
I can see there are not enough … so I need to exchange one … into ten …
is the whole … is a part ….is a part

Concrete and pictorial progression to support Mastery

Dexter is using place value counters to calculate 5,643 – 4,316
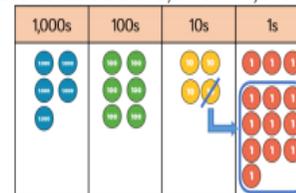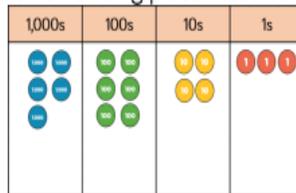
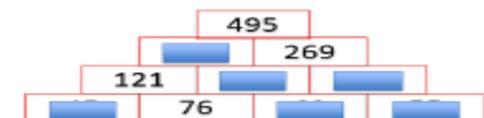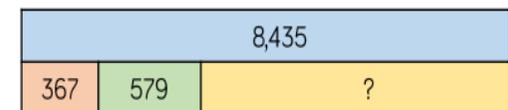**Concrete – use base ten and progress to place value counters**
Recognise the need to exchange - exchange - subtract   then   record

Multi-step problems

Subtraction – Progression in written methods – Stage 5

Key procedure:

Written column subtraction method with exchanging <u>across multiple place value columns</u> (5/6-digit numbers)

35003 - 24316

Sentence stems - Column Method (exchanging and unitising)

I can see there are not enough … so I need to exchange one … into ten …

13 ones subtract 6 one is 7 ones

9 tens subtract 1 ten is 8 tens

9 hundreds subtract 3 hundreds is six hundreds.

4 thousands subtract 4 thousands is zero.etc….

$$\begin{array}{r} ^4 3\cancel{5}\,^9\cancel{0}\,^9\cancel{0}\,^1 3 \\ -24316 \\ \hline 10687 \end{array}$$

Key skills:
- **Subtraction up to 6 digit numbers.**
- **Exchange multiple place values (understanding place value)**
- **Recognise and use the subtraction and equals symbol (-, =)**
- **Estimate answers and use the inverse to check**
- **Understand equality (=) and inequality (<,>)**

Key Vocabulary:
Subtract, take away, minus, decrease, fewer, difference, reduce, greater than, less than, inverse

Sentence stems:
how many fewer is…than..?, how much less is..? half, halve, how many are left?, one less, two less…, how many fewer is… than…?, how much less is…? =, equals, is the same as, count on, count back, how many more is…than..?
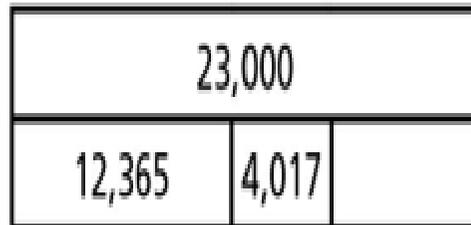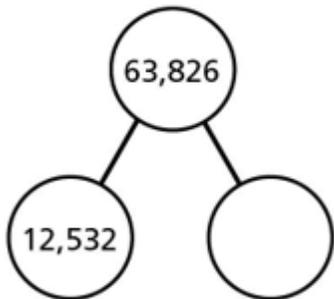I can see there are not enough … so I need to exchange one … into ten …
is the whole … is a part ….is a part

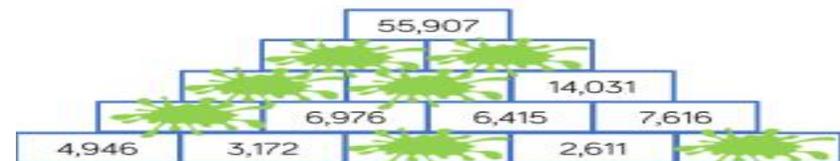**Concrete and pictorial progression to support Mastery**

Concrete – use place value counters

Pictorial

Multi-step problems

The sum of two numbers is 11,339
The difference between the same two numbers is 1,209
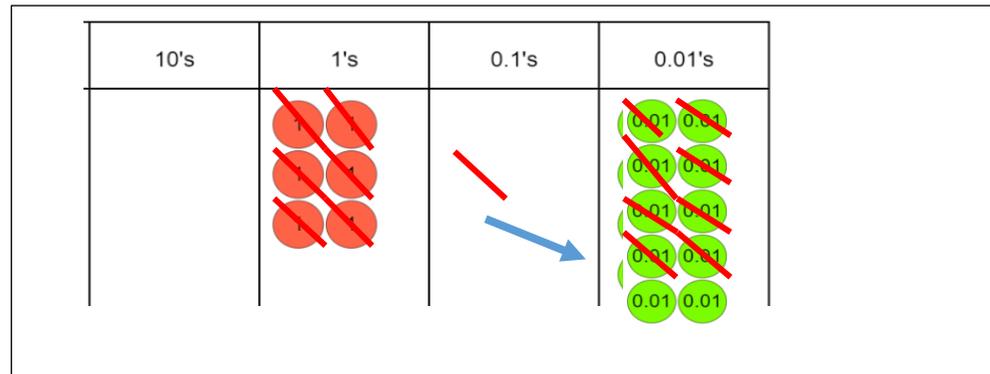Use the bar model to help you find the numbers.

Key procedure:

Formal column subtraction decimals with a different number of digits (including thousandths)

$$
\begin{array}{r}
57.\overset{2}{3}\overset{1}{0} \\
- \quad 6.08 \\
\hline
51.22 \\
\end{array}
$$
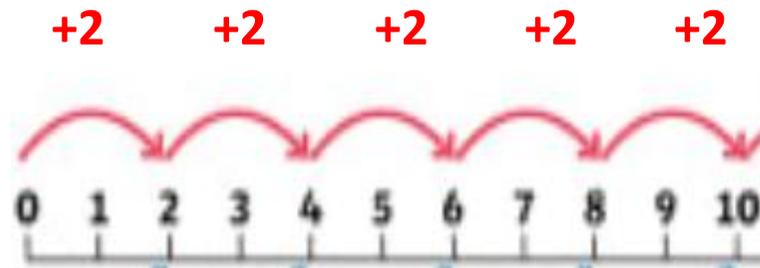
Key skill: Adding a place holder.

10's	1's	0.1's	0.01's

$$25.982 - 15.6 = \boxed{}$$

487.3	
?	2.9

Multiplication – Progression in written methods – Stage 1

|	
Key procedure: Use a number line to show repeated addition (multiplication) **+2 +2 +2 +2 +2** **5 groups of 2 make 10** **5 x 2 = 10**	**Key skills:** • **2, 5 and 10 times table** • **Understand multiplication as repeated addition** • **Recognise odd and even numbers** • **Recognise equal and unequal groups** • **Recognise and use the multiplication and equals symbol (x, =)** • **Understand that multiplication is commutative (e.g. 2x5 = 5x2)** **Key Vocabulary:** odd, even, count in twos, fives, count in tens (forwards from/backwards from), lots of, groups of, once, twice, five times, ten times, multiple of, times, multiply, multiply by, array, row, column, double. **Sentence stems:** … groups of … equals … … multiplied by… equals …

Concrete and pictorial progression to support Mastery
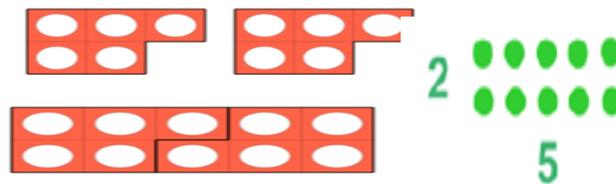
Concrete – use arrays/cubes/numicon/base ten
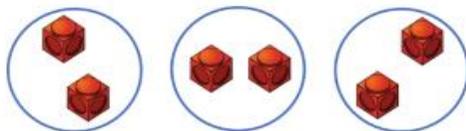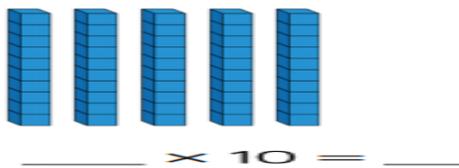
Pictorial
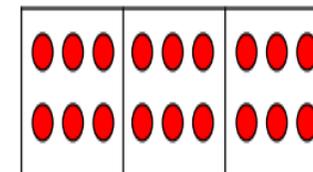
Multiplication – Progression in written methods – Stage 2

Key procedure:
Horizontal partitioning

12 x 3

 2 x 3 = 6

10 x 3 = 30

          36

Progress to short multiplication – no exchanging

$$12 \times 3 = 36$$

factor x factor = product

$$
\begin{array}{r}
1\ 2 \\
\times \quad\ 3 \\
\hline
3\ 6 \\
\hline
\end{array}
$$

Sentence stem
"3 groups of 2 ones is 6 ones.
3 groups of 1 ten is 3 tens.
3 tens added to 6 ones is 36.
The product of 12 and 3 is 36."

Key skills:
- **3, 4 and 8 times table**
- **Count in 3's, 4's, 8's. 50's and 100's**
- **2 digit by 1 digit multiplication**
- **Understand multiplication as repeated addition**
- **Recognise equal and unequal groups**
- **Recognise and use the multiplication and equals symbol (x, =)**
- **Understand that multiplication is commutative (e.g. 2x5 = 5x2)**

Key Vocabulary:
odd, even, count in twos/fives/ tens (forwards from/backwards from), lots of, groups of, once, twice, five times, ten times , multiple of, times, multiply, multiply by, array, row, column, double, product, factor, commutative, associative

Sentence stems: … is a factor … is a factor… is the product.
… groups of … ones is … ones.
… groups of … tens is … tens.
… tens added to … ones is … .
The product of … and … is … .

Concrete and pictorial progression to support Mastery
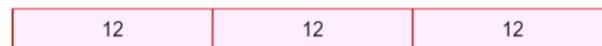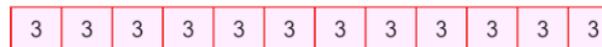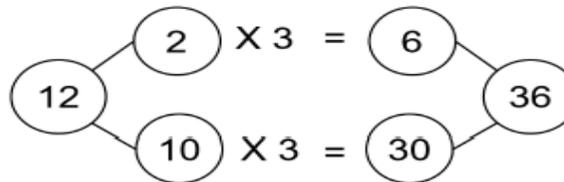**Concrete – use arrays/cubes/numicon/base ten**

Tens	Ones

12 x 3   3 x12

Pictorial-part whole/bar model

2 X 3 = 6

12

10 X 3 = 30

36

3	3	3	3	3	3	3	3	3	3	3	3

12	12	12

Problem solving

Complete the number sentences.

1 triangle has 3 sides.
3 triangles have ___ sides in total.
___ triangles have 6 sides in total.
5 triangles have ___ sides in total.

$1 \times 3 = 3$
$3 \times \_\_ = \_\_$
$\_\_ \times \_\_ = 6$
$\_\_ \times \_\_ = \_\_$

56				
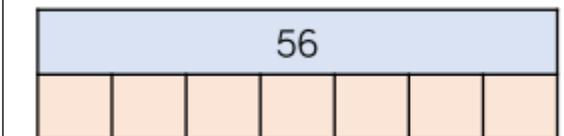
Multiplication – Progression in written methods – Stage 3

Key procedure:

Progress to short multiplication 3 digit by 1 digit (exchanging)

2 digit by 1 digit

$27 \times 5 = 135$

factor x factor = product

```
   2 7
 ×   5
 ─────
 1 3 5
   3
```

3 digit by 1 digit

$421 \times 3 = 1263$

```
   4 2 1
 ×     3
 ───────
 1 2 6 3
 1
```

Sentence stem:
3 groups of 1 one is 3 ones.
3 groups of 2 tens is 6 tens.
3 groups of 4 hundreds is 12 hundreds. I can **exchange** this for 1 thousand and 2 hundreds
The product of 421 x 3 is 1263.

Key skills:
- **All times tables up to 12x12**
- **Count in 6's, 7's, 9's. 25's and 1000's**
- **3 digit by 1 digit multiplication**
- **Recognise and use the multiplication and equals symbol (x, =)**
- **Understand that multiplication is commutative (e.g. 2x5 = 5x2)**
- **Understand that multiplication is distributive**
  (e.g. 6 x 4 = 6 x (3+1) = 6 x 3 + 6 x 1)

Key Vocabulary:
odd, even, count in twos/fives/ tens (forwards from/backwards from), lots of, groups of, once, twice, five times, ten times , multiple, times, multiply, multiply by, array, row, column, double, product, factor, commutative, associative, distributive
Sentence stems: … is a factor … is a factor… is the product.
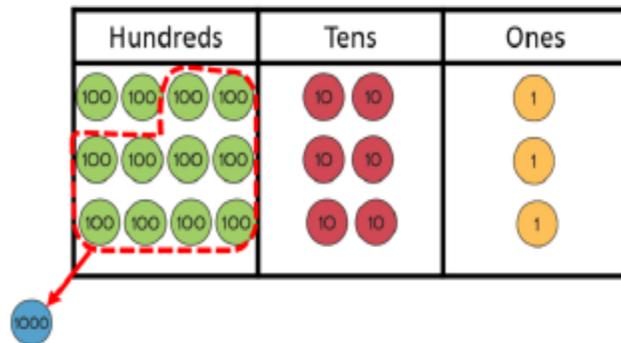… groups of … ones is … ones.
… groups of … tens is … tens.
… tens added to … ones is … .
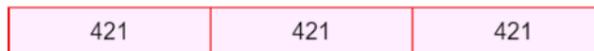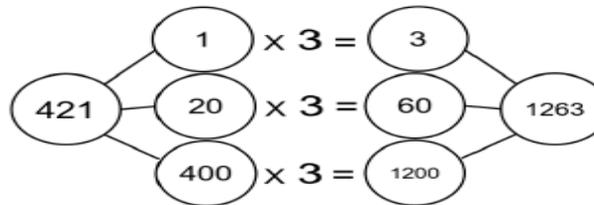The product of … and … is … .

Concrete and pictorial progression to support Mastery
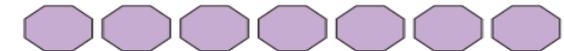**Concrete – use arrays/base ten/place value counters**

Pictorial-part whole/bar model

$1 \times 3 = 3$

$421 \quad 20 \times 3 = 60 \quad 1263$

$400 \times 3 = 1200$

421	421	421

Problem solving

There are ☐ hexagons.
There are ☐ sides on each hexagon.
☐ × ☐ = ☐
There are ☐ sides altogether.

$6 \times 2 = \underline{}$ $\underline{} \times 6 = 12$ $6 \times 2 \times 10 = \underline{}$

$\underline{} \times 20 = 120$ $20 \times \underline{} = 120$ $6 \times 2 \times \underline{} = 1,200$

$6 \times \underline{} = 1,200$ $200 \times 6 = \underline{}$ $10 \times \underline{} \times 6 = 120$

Multiplication – Progression in written methods – Stage 4

Key procedure:
Progress from long multiplication 2 by 2 digit to short multiplication 3 digit by 2 digit (exchanging) by year end.
2 digit by 2 digit (long)

32 x 14 = 448

```
    3  2
 x  1  4
 ───────
       8
    1  2  0
    1
       2  0
    3  0  0
 ─────────
    4  4  8
```

factor x factor = product

2 digit by 2 digit (short)

32 x 14 = 448

```
    3  2
 x  1  4
 ───────
    1  2  8
    1
    3  2  0
 ─────────
    4  4  8
```

Sentence stem:
4 groups of 2 ones is 8 ones.
3 groups of 3 tens is 12 tens.
I can __exchange__ this for 1 hundred and 2 tens…
The product of 32 x 14 is 448.

Key skills:
- **All times tables up to 12x12**
- **3 digit by 2 digit multiplication**
- **Understand factors, prime numbers, square and cubed numbers**
- **Understand that multiplication is commutative (e.g. 2x5 = 5x2)**
- **Understand that multiplication is distributive (e.g. 6 x 4 = 6 x (3+1) = 6 x 3 + 6 x 1)**

Key Vocabulary:
odd, even, lots of, groups of, once, twice, five times, ten times , multiple, times, multiply, multiply by, array, row, column, double, product, factor, commutative, associative, distributive, square number (^{2}), cubed number (^{3}), prime number, prime factor, composite (non-prime) number

Sentence stems: … is a factor … is a factor… is the product.
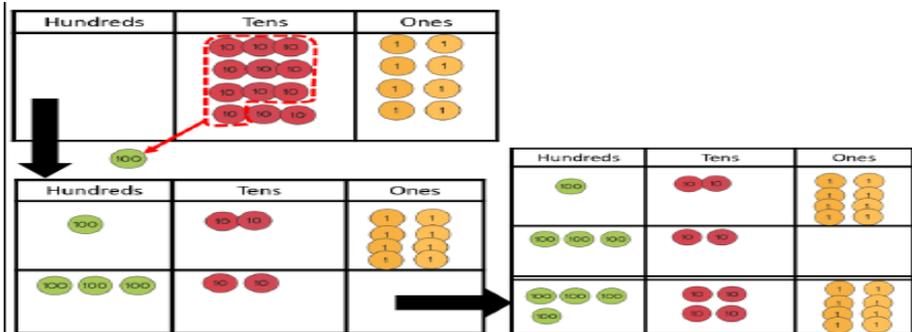… groups of … ones is … ones.
… groups of … tens is … tens.
… tens added to … ones is … .
The product of … and … is … .

Concrete and pictorial progression to support Mastery
Concrete – use arrays/base ten/place value counters

Pictorial-grid method to support place value

x	30	2	
10	300	20	= 320
4	120	8	= 128

Multiplication – Progression in written methods – Stage 4 contd.

Key procedure:
Progress from 2 by 2 digit to 3 digit by 2 digit (exchanging).

$132 \times 46 = 6{,}072$

factor x factor = product

```
  1 3 2
x   4 6
-------
  7 9 2
  1 1
5 2 8 0
1
6 0 7 2
  1 1
```

Sentence stem:
6 groups of 2 ones is 12 ones.
Exchange this for 1 ten and 2 ones.
6 groups of 3 tens is 18 tens. **Exchange** for 1 hundred and 8 tens…
The product of…

Key skills:
- **All times tables up to 12x12**
- **3 digit by 2 digit multiplication**
- **Understand factors, prime numbers, square and cubed numbers**
- **Understand that multiplication is commutative (e.g. 2x5 = 5x2)**
- **Understand that multiplication is distributive**
  (e.g. 6 x 4 = 6 x (3+1) = 6 x 3 + 6 x 1)

Key Vocabulary:
odd, even, lots of, groups of, once, twice, five times, ten times , multiple, times, multiply, multiply by, array, row, column, double, product, factor, commutative, associative, distributive, square number (^{2}), cubed number (^{3}), prime number, prime factor, composite (non-prime) number

Sentence stems: … is a factor … is a factor… is the product.
… groups of … ones is … ones.
… groups of … tens is … tens.
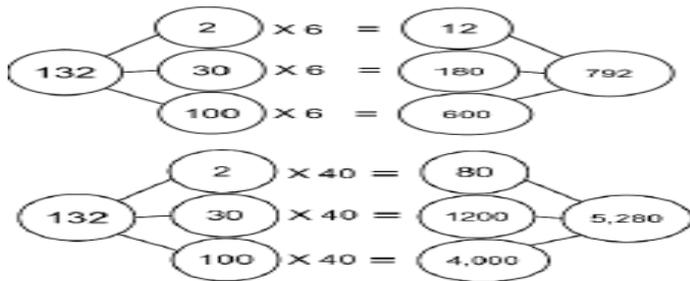… tens added to … ones is … .
The product of … and … is … .

Concrete and pictorial progression to support Mastery
Concrete – use arrays/base ten/place value counters

Pictorial-partitioning/grid method to support place value

x	100	30	2	
40	4000	1200	80	= 5280
6	600	180	12	= 792

Multiplication – Progression in written methods – Stage 5

Key procedure:
4 digit by 2 digit (exchanging).

4308 x 24

factor x factor = product

$$4308$$
$$x \quad 24$$
$$\overline{17232}$$
$$1\;1\quad\;3$$
$$86160$$
$$1$$
$$\overline{103,392}$$
$$1$$

Sentence stem:
6 groups of 2 ones is 12 ones.
Exchange this for 1 ten and 2 ones.
6 groups of 3 tens is 18 tens. Exchange for 1 hundred and 8 tens…
The product of…

Key skills:
- **All times tables up to 12x12**
- **4 digit by 2 digit multiplication**
- **Multiply decimals up to 2 decimal places by a whole number**
- **Understand factors, prime numbers, square and cubed numbers**
- **Understand that multiplication is commutative (e.g. 2x5 = 5x2)**
- **Understand that multiplication is distributive**
  (e.g. 6 x 4 = 6 x (3+1) = 6 x 3 + 6 x 1)

Key Vocabulary:
odd, even, lots of, groups of, once, twice, five times, ten times , multiple, times, multiply, multiply by, array, row, column, double, product, factor, commutative, associative, distributive, square number (^{2}), cubed number (^{3}), prime number, prime factor, composite (non-prime) number
Sentence stems: … is a factor … is a factor… is the product.
… groups of … ones is … ones.
… groups of … tens is … tens.
… tens added to … ones is … .
The product of … and … is … .

Concrete and pictorial progression to support Mastery
Concrete – use arrays/base ten/place value counters
Pictorial-partitioning/grid method to support place value

x	4000	300	8	
20	80000	6000	160	= 86160
4	16000	1200	32	= 17232

Multiplication – Progression in written methods – Stage 5 contd.

Key procedure:
Multiply decimals by a whole number

$$34.2 \times 6 = 205.2$$

```
 3 4 . 2
x    6
---------
2 0 5 . 2
---------
  1 2 1
```

Sentence stem:
6 groups of 2 tenths is 12 tenths…
Exchange this for 1 one and 2 tenths.
…
The product of…

Key skills:
- **All times tables up to 12x12**
- **4 digit by 2 digit multiplication**
- **Multiply decimals up to 2 decimal places by a whole number**
- **Understand factors, prime numbers, square and cubed numbers**
- **Understand that multiplication is commutative (e.g. 2x5 = 5x2)**
- **Understand that multiplication is distributive**
  (e.g. 6 x 4 = 6 x (3+1) = 6 x 3 + 6 x 1)

Key Vocabulary:
odd, even, lots of, groups of, once, twice, five times, ten times , multiple, times, multiply, multiply by, array, row, column, double, product, factor, commutative, associative, distributive, square number (^{2}), cubed number (^{3}), prime number, prime factor, composite (non-prime) number

Sentence stems: … is a factor … is a factor… is the product.
… groups of … ones is … ones.
… groups of … tens is … tens.
… tens added to … ones is … .
The product of … and … is … .

Concrete and pictorial progression to support Mastery
Concrete – use arrays/base ten/place value counters
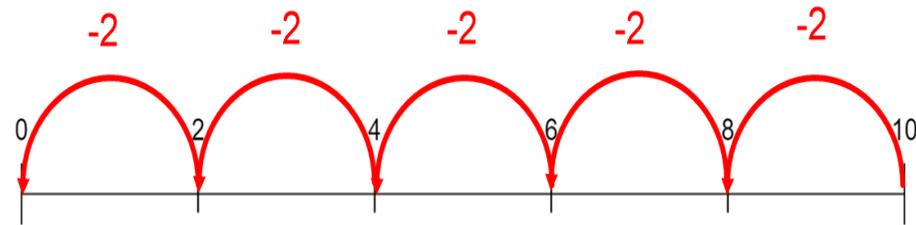
Pictorial-partitioning/grid method to support place value

x	30	4	0.2
6	180	24	1.2

= 205.2

Division – Progression in written methods – Stage 1

Key procedure:

Use a number line to show repeated subtraction (division)

$$10 \div 5 = 2$$

dividend ÷ divisor = quotient

Key skills:
- **Recognise odd and even numbers/groups**
- **Halving**
- **Division facts linked to 2,5 and 10 times table**
- **Understand that division is NOT commutative (e.g. 6÷2 is not equal to 2÷6)**

Key Vocabulary:
groups of, equal groups of, odd, even. halve, share, share equally, divide, divided by, divided into, repeated subtraction, inverse

Sentence stems:
Share … equally between… (sharing structure)
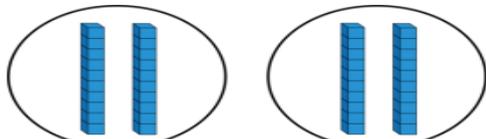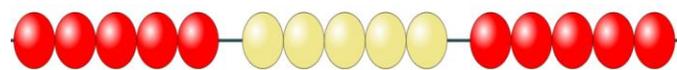How many equal groups can you make? (grouping structure)
How many groups of …are there in…?

Concrete and pictorial progression to support Mastery
Concrete – use arrays/base ten/bead strings/numicon
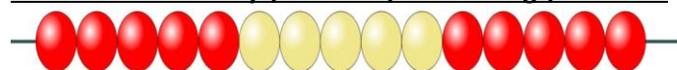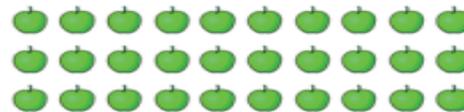

15 ÷ 3 = 5

40 ÷ 2 = 20

Apples can be sold in packs of 10
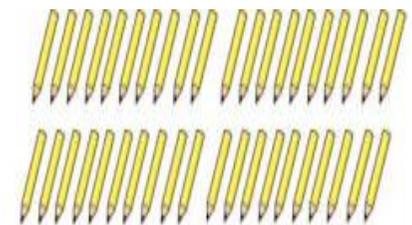How many packs can be made below?

Which number pieces are odd? Explain why.
Find or draw other odd and even pieces.
What do you notice?

40 pencils are shared between 5 children.

How many pencils does each child get?

Division – Progression in written methods - Stage 2

Key procedure:
Long division 2 digits ÷ 1 digit (progress to exchanging).

$$74 \div 2 = 37$$

quotient
divisor)dividend

NB – Jottings of the divisor can be useful.
2
4
6
8
10
12 …

$$2 \overline{)74}^{1}$$
37

Sentence stem:
First share 7 tens into 2 equal groups.
There are 3 tens in each group.
I have 1 ten remaining.
I need to exchange the remaining 1 ten into 10 ones.
I now have 14 ones in total.
Then, I am sharing 14 ones into 2 equal groups.
There are 7 ones in each group.
The quotient (answer) is 37

NB- always refer to place value.

Key skills:
- **Recognise odd and even numbers/groups**
- **Division facts linked to 3,4 and 8 times table**
- **Understand that division is NOT commutative (e.g. 6÷2 is not equal to 2÷6)**
- **Divide 2 digit numbers by 1 digit numbers**

Key Vocabulary:
groups of, equal groups of, odd, even. halve, share, share equally, divide, divided by, divided into, repeated subtraction, inverse
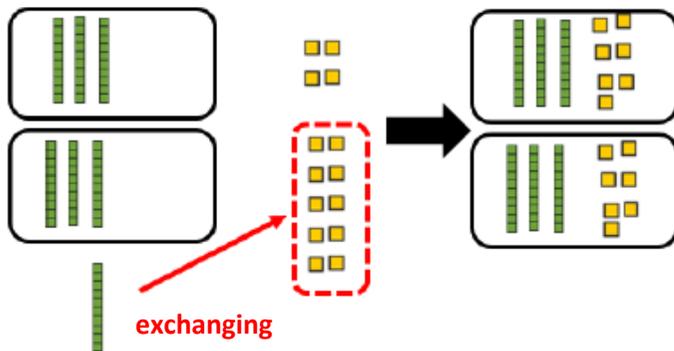
Sentence stems:.
Share … equally between… (sharing structure)
How many equal groups can you make? (grouping structure)
How many groups of …are there in…

Concrete and pictorial progression to support Mastery
Concrete – use arrays/base ten/bead strings/counters

exchanging

Circle the counters in groups of 3 and complete the division.

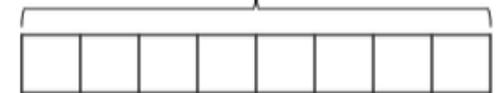___ ÷ 3 = ___

Circle the buttons in groups of 4.

Can you also split the buttons into 4 equal groups?
How is this the same? How is it different?

Amir shares 24 sweets equally between 8 friends.
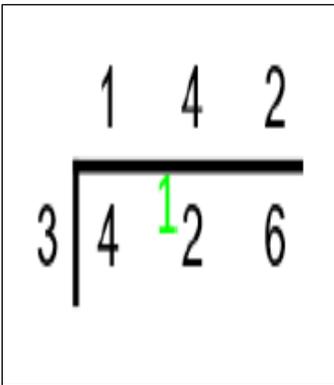How many do they get each?
24

Division – Progression in written methods – Stage 3

Key procedure:
Short division 3 digit ÷ 1 digit (introduce remainders).

$426 \div 3 = 142$

quotient
divisor)dividend

Example **with remainder**:

$376^{r\,1}$
$2\overline{)753}$

NB- **always** refer to place value.

Sentence stem:
First share 4 hundreds into 3 equal groups.
There is 1 hundred in each group.
I have 1 hundred remaining.
I need to exchange the remaining 1 hundred into 10 tens.
I now have 12 tens in total.
Then, I am sharing 12 tens into 3 equal groups.
There are 4 tens in each group.
Share 6 ones into 3 equal groups. There are 2 ones in each group.
The quotient (answer) is 142

Key skills:
- **Division facts linked up to 12x table**
- **Divide 3 digit numbers by 1 digit numbers (with remainders)**

Key Vocabulary:
groups of, equal groups of, odd, even. halve, share, share equally, divide, divided by, divided into, repeated subtraction, inverse, factor, factor pair, divisor, divided by, divided into, remainders, divisible by, equivalent, short division, quotient, remainder, multiples, exchange.

Sentence stems:.
Share … equally between… (sharing structure)
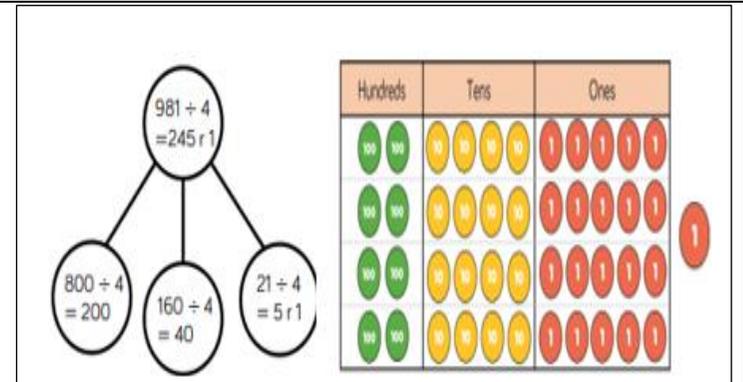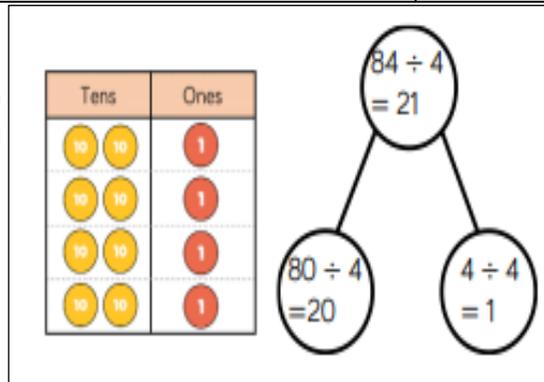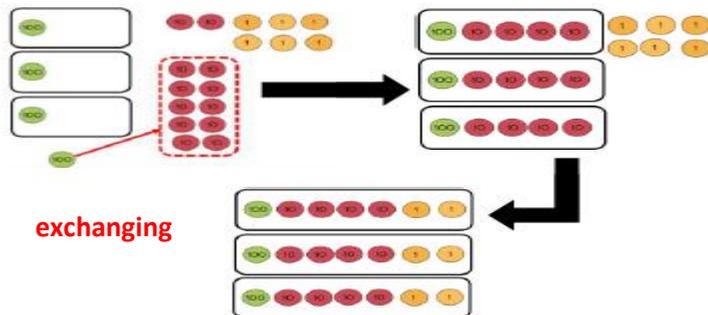How many equal groups can you make? (grouping structure)
How many groups of …are there in…?

Concrete and pictorial progression to support Mastery
Concrete – use arrays/base ten/bead strings/counters

exchanging

Division – Progression in written methods – Stage 4

Key procedure:
Short division up to
4 digit ÷ 1 digit
(with remainders)

$$438 \div 3 = 146$$

quotient
divisor)dividend

NB- **always** refer to place value.

Sentence stem:
First share 4 hundreds into 3 equal groups.
There is 1 hundred in each group.
I have 1 hundred remaining.
I need to exchange the remaining 1 hundred into 10 tens.
I now have 13 tens in total.
Then, I am sharing 13 tens into 3 equal groups.
There are 4 tens in each group.
I have 1 ten remaining…
The quotient (answer) is 146

Key skills:
- **Division facts linked up to 12x table**
- **Understand that division is NOT commutative (e.g. 6÷2 is not equal to 2÷6)**
- **Divide 4 digit numbers by 1 digit numbers**
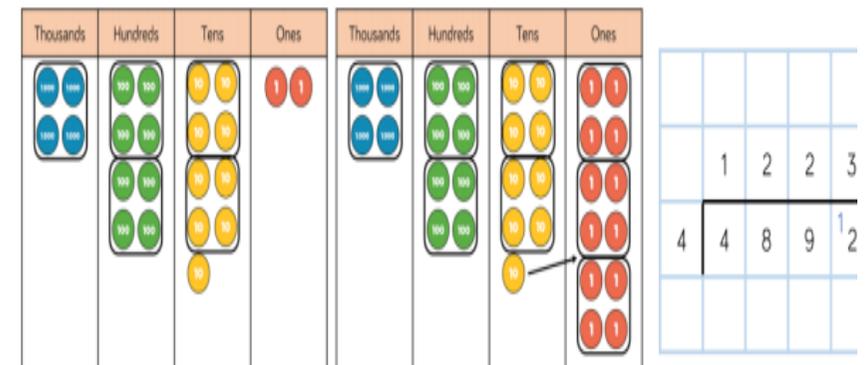- **Interpret remainders in context**

Key Vocabulary:
groups of, equal groups of, odd, even. halve, share, share equally, divide, divided by, divided into, repeated subtraction, inverse, factor, factor pair, divisor, divided by, divided into, remainders, divisible by, scaling, equivalent, short division, quotient, remainder, multiples, exchange.

Sentence stems:.
Share … equally between… (sharing structure)
How many equal groups can you make? (grouping structure)
How many groups of …are there in…?

Concrete and pictorial progression to support Mastery
Concrete – use arrays/base ten/counters

Division – Progression in written methods – Stage 4 contd.

Key procedure:
Short division up to 4 digit ÷ 1 digit
(fraction remainders)

$$75 \div 6 = 12\frac{1}{2}$$

$$\frac{\text{quotient}}{\text{divisor }) \text{ dividend}}$$

$$6 \overline{\smash{\big)}\ 7\ ^{1}5} \quad 1\quad 2\ ^{\frac{1}{2}}$$

The remainder of 3 is 3 out of 6 (6 being the divisor).

This is simplified to 1 —
2

Key skills:
- **Division facts linked up to 12x table**
- **Understand that division is NOT commutative (e.g. 6÷2 is not equal to 2÷6)**
- **Divide 4 digit numbers by 1 digit numbers**
- **Interpret remainders in context**
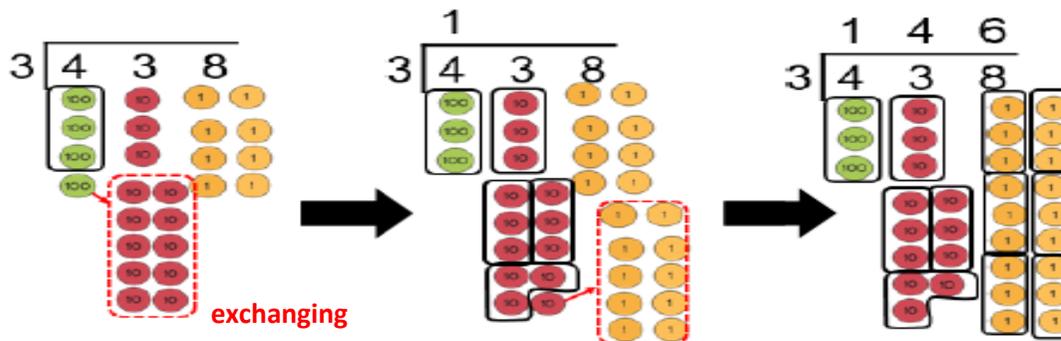
Key Vocabulary:
groups of, equal groups of, odd, even. halve, share, share equally, divide, divided by, divided into, repeated subtraction, inverse, factor, factor pair, divisor, divided by, divided into, remainders, divisible by, scaling, equivalent, short division, quotient, remainder, multiples, exchange.

Sentence stems:.
Share … equally between… (sharing structure)
How many equal groups can you make? (grouping structure)
How many groups of …are there in…?

Concrete and pictorial progression to support Mastery
Concrete – use arrays/base ten/counters

Division – Progression in written methods – Stage 4 contd.

Key procedure:
Short division up to 4 digit ÷ 1 digit
(decimal remainders)

$$75 ÷ 6 = 12.5$$

quotient
––––––
divisor)dividend

1 2 .5
6) 7 ¹5 .0

Sentence stem:
First share 7 tens into 6 equal groups.
There is 1 ten in each group.
I have 1 ten remaining.
I need to exchange the remaining 1 ten into 10 ones.
I now have 15 ones in total.
Then, I am sharing 15 ones into 6 equal groups.
There are 2 ones in each group.
I have a remainder of three. I need to exchange 3 ones into 30 tenths.
NB- **always** refer to place value.

Key skills:
- **Division facts linked up to 12x table**
- **Understand that division is NOT commutative (e.g. 6÷2 is not equal to 2÷6)**
- **Divide 4 digit numbers by 1 digit numbers**
- **Interpret remainders in context**

Key Vocabulary:
groups of, equal groups of, odd, even. halve, share, share equally, divide, divided by, divided into, repeated subtraction, inverse, factor, factor pair, divisor, divided by, divided into, remainders, divisible by, scaling, equivalent, short division, quotient, remainder, multiples, exchange.
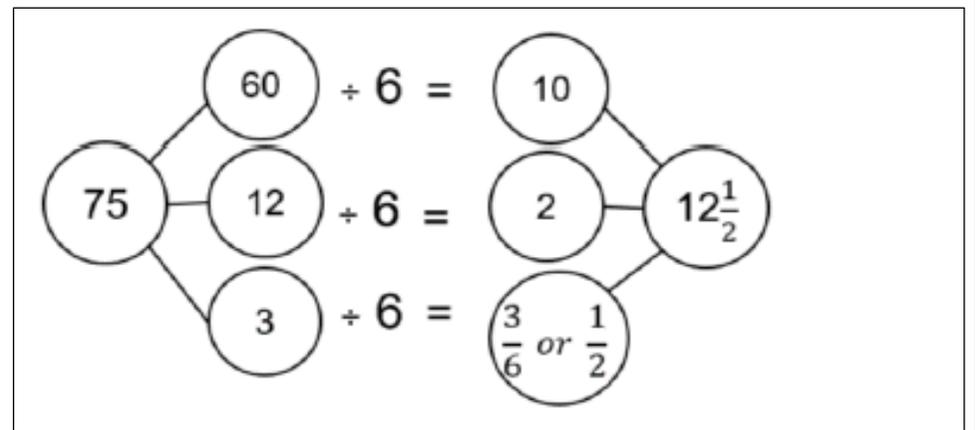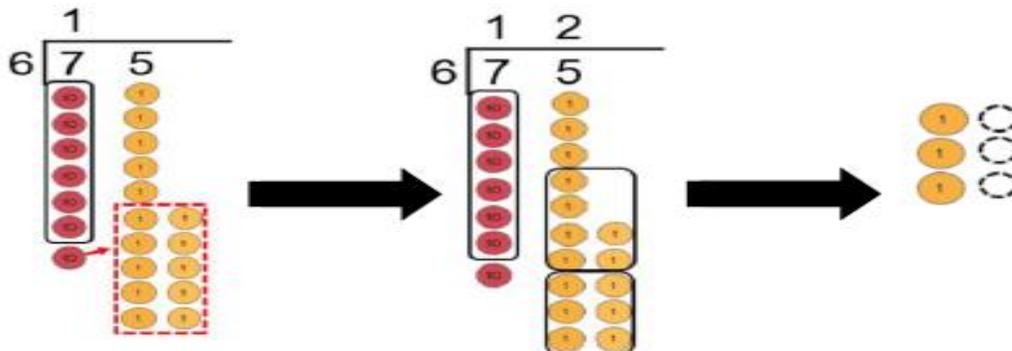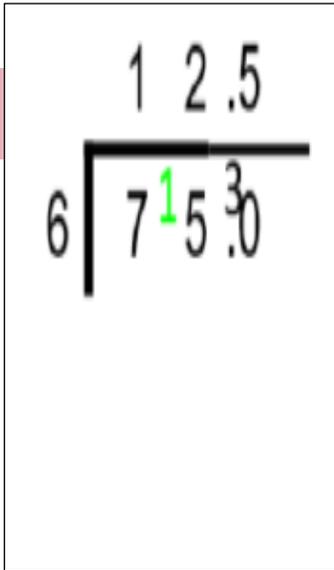
Sentence stems:.
Share … equally between… (sharing structure)
How many equal groups can you make? (grouping structure)
How many groups of …are there in…?

Concrete and pictorial progression to support Mastery
Concrete – use arrays/base ten/counters

Jottings: multiples of the divisor

6
12
18
24
30
36
42
48
54
60

Division – Progression in written methods – Stage 5

Key procedure:	Key skills:
Long division up to 4 digit ÷ 2digit NB – Long division only for 2 digit divisors.	- **Division facts linked up to 12x table** - **Understand that division is NOT commutative (e.g. 6÷2 is not equal to 2÷6)** - **Divide 4 digit numbers by 2 digit numbers** - **Interpret remainders in context**

$$3016 \div 13 = 232$$

Jottings: multiples of the divisor
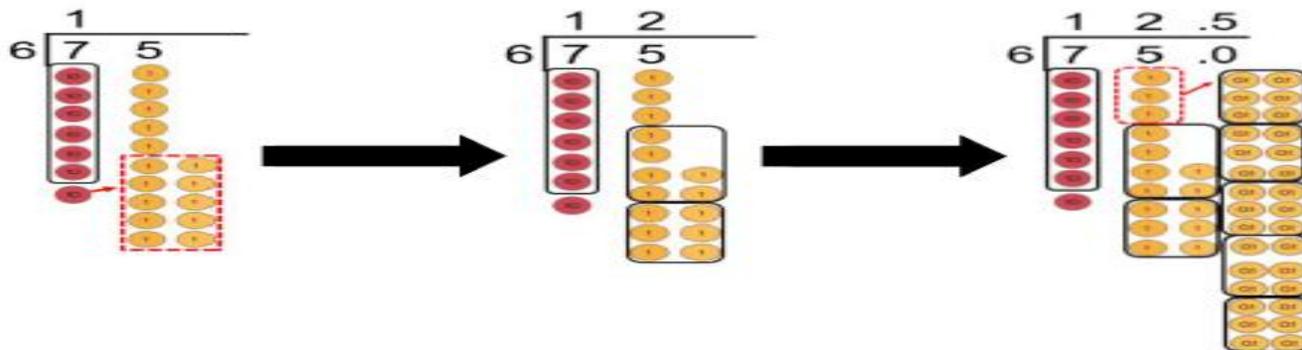
13
26
39
52
65
78
91
104

Key Vocabulary:

groups of, equal groups of, odd, even. halve, share, share equally, divide, divided by, divided into, repeated subtraction, inverse, factor, factor pair, divisor, divided by, divided into, remainders, divisible by, scaling, equivalent, long/short division, quotient, remainder, multiples,

Sentence stems:.

Share … equally between… (sharing structure)
How many equal groups can you make? (grouping structure)
How many groups of …are there in…?

Concrete and pictorial progression to support Mastery

Concrete – use arrays/base ten/counters

Appendix 1. Glossary of terms

Exchange - Change a number for another of equal value. The process of exchange is used in some standard compact methods of calculation.

Partition(ing) - To split a number into component parts. Example: the two-digit number 38 can be partitioned into 30 + 8 or 19 + 19.

Commutative - Addition and multiplication of real numbers are commutative where a + b = b + a and a × b = b × a.

Dividend - In division, the number that is divided.

Divisor - The number by which another is divided.

Quotient - The result of a division.